The Journal of

THE ROYAL AERONAUTICAL SOCIETY

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NOVEMBER 1947 NO 443 · VOL. 51

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NOVEMBER NOTICES 1947

JOURNAL PREMIUM AWARDS

The Council have set aside an annual sum of £250 for the award of premiums for papers published in the *Journal* and the Council hope that members (or non-members) will contribute papers on their own special subjects.

CANCELLED LECTURE

The joint meeting arranged with the Institution of Mechanical Engineers for 28th November 1947 at the Institution has been postponed.

The meeting was to hear a paper by Mr. W. H. Lindsey on " The Development of the A.S. Type Gas Turbine," but the Ministry concerned has not found it possible to release, in time for the lecture, much of the information upon which the lecture was based.

CONTENTS OF NOVEMBER JOURNAL

Minimising Zero Drift in Electrical Strain Gauge Bridges, by C. R. Urwin, A.C.G.I., A.M.I.E.E., and Dr. K. H. Swainger, Ph.D., B.Sc.

Influence of Aircraft Gross Weight on the Size and Weight of Hulls and Fuselages, by L. W. Rosenthal, A.F.R.Ae.S.

The General Theory of Cylindrical and Conical Tubes under Torsion and Bending Loads —Part V, by J. Hadji-Argyris, D.E., A.F.R.Ae.S., and P. C. Dunne, B.Sc., A.F.R.Ae.S.

Activities of the Society—Aeronautical Conference, Guildhall Dinner and Garden Party.

CIERVA MEMORIAL PRIZE ESSAY

The closing date for entries for the 1947/48 Cierva Memorial Prize Essay is the 31st January 1948. This prize is open to competitors of all nationalities and is not confined to members of the Helicopter Association of Great Britain. Competitors must be under the age of 35 on the 31st December 1947.

Copies of the rules and conditions for the 1947/48 competition may be had from the Honorary Secretary, The Helicopter Association of Great Britain, Finsbury Circus House, Blomfield Street, London, E.C.2.

WOMEN'S LEGION SCHOLARSHIP IN AERONAUTICAL ENGINEERING

A new scholarship for women in aeronautical engineering has been founded by The Women's Legion and The Women's Engineering Society. The scholarship will be awarded every two years and the winner will be able to train in any branch of Aeronautical Engineering. Girls who have passed the School Certificate Examination and are between 15 and 20 years of age are eligible. Applications for the first scholarship must be received before the 30th April 1948.

The new scholarship owes its origin to the generosity of The Women's Legion which was founded during the 1914-18 War and continued its activities during the recent war. The Legion now feels that its services are no longer actively needed and has decided to use the funds at its disposal to aid the peace-time training of women. A Trust Fund and the

scholarship have been created for this purpose. The administration of the scholarship has been undertaken by The Women's Engineering Society, founded in 1919.

Application forms for the scholarship may be obtained from The Secretary, The Women's Engineering Society, 35 Grosvenor Place, London, S.W.1, to whom all enquiries should be made.

" PROBLEMS FACING CIVIL AIR OPERATIONS " by N. E. Rowe, Esq., C.B.E., B.Sc., D.I.C., F.R.Ae.S.—*Summary of the lecture to be given on 4th December* 1947.

The lecture takes three broad problems, each raising a number of subsidiary problems, which face airline operations now and will continue to offer difficulties in the longer term future.

Various aspects of the problems and possible solutions are discussed.

The main and subsidiary points to be covered are:—

(1) Provision of reliable and regular service in all reasonable conditions of weather and climate; involving solution of navigation and traffic control problems, improvement of mechanical reliability, the development of aids to approach and landing in low visibility conditions, problems of pilot fatigue, and so on.

(2) The economical achievements of higher standards of safety, *i.e.*, safety with economy; covering the problem of division of responsibility between operators and regulatory authorities, air traffic control and overall economy, standards and methods of operation, and so on.

(3) The reduction of the cost of air transport as affected by the aircraft to be operated and their best use.

PRINCIPAL SCIENTIFIC OFFICER

Applications are invited by the Civil Service Commissioners for the post of Principal Scientific Officer for Aircraft Structural Research in the Royal Aircraft Establishment, South Farnborough, Hants.

Candidates must be British subjects, born on or before 1st August 1915. They should have an honours degree, preferably in engineering, and have had experience in design stressing or weight analysis in the Aircraft Industry.

The successful candidate will be required to supervise research work to improve the structural efficiency of future aircraft and must also be prepared to do original research.

The appointment is permanent, with superannuation benefits under the Federated Superannuation System for Universities and is graded as Principal Scientific Officer on the inclusive scales £860 to £1,155 (men), £760 to £997 (women).

Forms of application are obtainable from the Secretary, Civil Service Commission, Scientific Branch, 27 Grosvenor Square, London, W.1, quoting No. 2019, to whom completed forms must be returned **not later than 1st December, 1947.**

LECTURE PROGRAMME—AUTUMN SESSION 1947-1948

The Lectures will be held at 6 p.m. in the Lecture Hall of the Institution of Civil Engineers, Great George Street, S.W.1 (by permission of the Council of the Institution) unless otherwise stated. Tea will be served at 5.30 p.m.

Visitors are welcome, but should obtain tickets through a member of the Society.

Thursday, 6th November 1947—Some recent Developments in the Landing Gear Field by Captain R. Lucien, A.F.R.Ae.S.

Thursday, 4th December 1947—Problems Facing Civil Air Operations, by N. E. Rowe, C.B.E., B.Sc., D.I.C., F.R.Ae.S.

Thursday, 18th December 1947—The Work of the High-Speed Tunnel, by Professor A. Thom, M.A., D.Sc., and W. G. A. Perring, F.R.Ae.S.

Thursday, 15th January 1948—Gas Turbine Accessory Systems, by O. Lawrence, A.F.R.Ae.S. JOINT MEETING WITH THE INSTITUTE OF FUEL.

Thursday, 29th January 1948—Flight Testing of Helicopters, by W. Stewart, B.Sc.

BELFAST BRANCH

Tuesday, 18th November 1947—Bird Flight and the Aeroplane, Captain J. L. Pritchard, Hon. F.R.Ae.S., F.I.Ae.S.

Tuesday, 9th December 1947—Title not yet agreed, Sir John Buchanan, C.B.E., F.R.Ae.S., A.M.I.Mech.E.

Tuesday, 6th January 1948—Some Experiences in Experimental Gliding, Prof. Hill, M.C., M.Sc., M.I.Mech.E., F.R.Ae.S.

Friday, 23rd January 1948—Smoking concert.

Tuesday, 3rd February 1948—High Speed Flight, Dr. Hilton.

Tuesday, 24th February 1948—Evening of lecturettes.

Tuesday, 16th March 1948—Inside Occupied Japan, Mr. William Courtenay, M.M., A.R.Ae.S.

All meetings are held at the Assembly Hall, Belfast College of Technology, at 7.30 p.m.

BIRMINGHAM BRANCH

Friday, 14th November 1947—Selection of Power Plant for Aircraft, by E. C. J. O'Hanlon, A.F.R.Ae.S.

At the Birmingham Chamber of Commerce, New Street, at 7.30 p.m.

BRISTOL BRANCH

Wednesday, 12th November 1947—The Production Problem, by F. H. Parker, A.M.I.P.E., Chief Production Engineer, Bristol Aeroplane Co. Ltd., Aircraft Division.

Tuesday, 25th November 1947—The Preparation of an Engineer, by W. W. Evans, B.A., B.Sc., Principal, Miles Technical School.

Wednesday, 10th December 1947—Air Crew Training, by Captain A. C. P. Johnstone, O.B.E., A.F.R.Ae.S., F.R.Met.S., A/Manager, Airways Training Ltd. (Subject to his being available at this date. Members will be informed by post.)

Tuesday, 20th January 1948—Debate.

Wednesday, 11th February 1948—The Design of a Large Naval Vessel, by H. S. Pengelley, R.C.N.C., M.Inst.N.A., Assistant Director of Naval Construction, Admiralty.

Monday, 23rd February 1948—Accidents and Their Investigation, by Air Commodore Vernon Brown, C.B., O.B.E., M.A., F.R.Ae.S., Accidents Investigation Branch, Ministry of Civil Aviation.

Wednesday, 3rd March 1948—Reading of Prize-winning Papers by Junior Members.

Monday, 22nd March 1948—Operation of Civil Transport Aircraft, by N. E. Rowe, Esq., C.B.E., F.R.Ae.S., Controller of Research and Long Term Development, British European Airways.

Wednesday, 14th April 1948—Film Night and Annual General Meeting.

All meetings will be held in the Conference Room, Bristol Aeroplane Co. Ltd., Filton House, at 6.0 p.m.

GLOUCESTER AND CHELTENHAM BRANCH

Thursday, 20th November 1947—Re-fuelling in the Air (with film), W. Compton-Holgate—at Gloucester.

Wednesday, 10th December 1947—Rotor Systems and Control Problems in the Helicopter, Raoul Hafner, Chief Designer, Helicopter Department, Bristol Aeroplane Co. — at Cheltenham.

Thursday, 15th January 1948—Film on Rocket Flight (Sound track in English)—at Gloucester.

Wednesday, 4th February 1948—Air Operations in the Battle of Britain, An Officer in the Royal Air Force—at Cheltenham.

Thursday, 4th March 1948—The Development of the A.S. Type Gas Turbine Engine, W. H. Lindsey, M.A., A.F.R.Ae.S.—at Gloucester.

Wednesday, 7th April 1948—Annual General Meeting followed by Film—at Cheltenham.

Gloucester meetings are held in the Wheatstone Hall, City Library, Brunswick Road, Gloucester, at 7.30 p.m. on *Thursdays*.

Cheltenham meetings are held in the Chemistry Lecture Theatre, Grammar School, Cheltenham, at 7.30 p.m. on *Wednesdays*.

LUTON BRANCH

Wednesday, 12th November 1947—Air Safety, by Group Captain Saker, M.V.O., A.F.R.Ae.S.

Wednesday, 3rd December 1947—Aerodrome Construction, by Brian H. Colquhoun, B.Sc., M.Inst.C.R., M.I.Strut.E., M.Soc.Ing., Civ.De.Fr.

All meetings will be held at The George Hotel, at 7. p.m.

PRESTON BRANCH

Wednesday, 19th November 1947—The Probable Role and Influence of Aircraft in Future Warfare, by Air Marshal Sir Robert H. M. S. Saundby, K.B.E., C.B., M.C., D.F.C., A.F.C

Wednesday, 10th December 1947—Fighter Design, by Professor R. L. Lickley, B.Sc., F.R.Ae.S. (Professor of Aircraft Design at the College of Aeronautics, Cranfield).

Unless otherwise stated, all lectures will be held at 7 p.m. in the Preston Chamber of Commerce, Fishergate (next to the Theatre Royal).

SOUTHAMPTON BRANCH

Wednesday, 3rd December 1947—Recent Development in Flying-Boats, by H. Knowler, A.F.R.Ae.S.

Wednesday, 7th January 1948—Rotating Wing Aircraft, by C. G. Pullin.

Wednesday, 4th February 1948—Structures Testing by P. B. Walker, M.A., Ph.D., F.R.Ae.S.

Lectures will be held at University College, Southampton, at 7 p.m.

GRADUATES' AND STUDENTS' SECTION

Wednesday, 19th November 1947—Aircraft Photography, by John Yoxall, Art Editor of *Flight*.

Friday, 12th December 1947—Aircraft Design from the Airline Point of View, by Christopher Dykes, Deputy Chief Project Engineer, B.O.A.C.

Lectures will be held in the Library of the Society, 4 Hamilton Place, W.1, at 7.30 p.m.

CHANGES OF ADDRESS

To assist in keeping the records of members correct and up to date the Secretary will be glad if all members will notify him as soon as possible of changes of address.

When notifying changes please give the following particulars:—

Name (in block letters).
Grade of membership.
New address (in block letters).
Old address.

Changes of address must be received *before the 15th of the month* in order to be effective for the Journal for the following month.

ELECTION OF MEMBERS

The following new members have been elected:—

Associate Fellows

George Murison Addison (from Graduate), Francis William Northcote Bassett, Leonard Ambrose Beale, Basil Davenport Blackwell, Thomas Henry Blake, Lewis Boddington, Stanley Haslam Bonser, John Longmuir Penman Brodie, Guy Birch Brushfield (from Graduate), Edwin James Catt (from Associate), Thomas Clapham, Sidney Arthur Clarke, Whitworth Thomas George Collyer, Frank Coulthurst, Frank Turner Dent (from Graduate), Padriac Cathal Dunne, Hubert Thomas Ellingworth, Samel Farquar, Frederick Farrington, Gawaine Gerald Gallaher, Jack Hall, William Lyddon Harper, Edward Peterson Hawthorne, James Carson Howard, John Hunter Hunter-Tod, Anne Mary Hyde, George William Eric Jameson, George Alec Jones (from Graduate), John Harold Parry Jones, William Aylwin Jones (from Graduate), William Alfred Clifford Kendall (from Graduate), Douglas John Kenney, William Taylor Kirkby (from Graduate), John Gareth Frank Lang (from Graduate), George Berrell Leather, Robert Norman Lindley, Pierce McDiarmid, Edward Thomas Marles, Arthur Reginald May, Peter Richard Mersy, Francis William Morgan (from Graduate), George Albert Naylor, Louis Paul O'Connor, John Edward Pipe, Charles Norman Seton Pringle, Nathan Owen Rawlinson (from Graduate), David George Aidan Rendel (from Graduate), Gordon Richmond, Maurice Rothman (from Graduate), Arthur Leslie Ruckley, Ian Lascelles Scott-Buccleuch, Julian Jay Sharpe (from Associate), Brian Hudson Slatter, Robert Arthur Smith, Albert Richard Eric Stevens, Herbert Titmuss, Arthur Walter Turner (from Graduate), Dennis Blackwell Wallis (from Graduate), Oscar Watson (from Graduate), Frederick William Whitehead, Benjamin Jones Williams, William Young Wilson (from Graduate), Frederick Peter Youens (from Graduate).

Associates

Alfred James Andrew Armstrong, Harold Robert Bunn, Robert Albert Carr-Lewty, Thomas Dorian Robertson Carroll (from Student), Edward Desmond Cecil Cooper, Douglas Dewar, Neville John Ebsworth (from Student), Albert Victor Faure, Douglas Oscar Fraser, Peter John Garner, Edward Garden Hopcraft, Clive Lawson Hullock, Charles Herbert Hyde, Kenneth Paton Jones, T. S. Krishnamurthy, John Oliver Lancaster, Hubert John Langtry, Gordon Medd, Deena Dayalam Namsoo, Cyril Bernard Horace Osborne, William Bryan Richardson (from Student), John Brickwood Waite (from Student).

Graduates

Peter Robert Allen, David Burns (from Student), David John Caple, James Roy Combley (from Student), Frederick John Ernest Davison (from Student), Harold Beaton

Grant, Leonard Desmond Gurney, Derek Evan Hammond, Joseph Edward Hart, Newton David Hodgekiss, Cecil Henry Jones, Charles Neville Jones (from Student), Samuel Hadyn Jones, Walter Sydney Jones, Bernard Lovegrove (from Student), John Mackereth (from Student), Anthony John Monk, William Edward Morris, Lawrence Wynn Richards, Ronald Alfred Smithers (from Student), Roman Wladyslaw Szymanski, John Graham Walker, Peter Alan Lennox Watson, George Young.

Students

Zena Addlestone, Clifford Lester Bore, Geoffrey Bradman, Harold Caplan, Leslie Maurice Carlile, David Anthony Doidge, George Herbert Fenton, Robert John Gladwell, Edward David Hibberd Hall, Donald William Hayward, Henry Anthony Knight, John Freeman Leach, William MacPherson, Richard John Margetts, Tony Henry Marsden-Jones, Michael Nowell Morss, Gerhard Platau, John Harry Norman Tye, Gerald David Walley, Ian Frederick Watson, Rex Maddock Whalley, Winstone Harold Williamson, John Charles Winterburgh.

Companions

Peter George Griffiths (from Student), Charles Alec Guy Hicks, Robert Murdoch Wilson (from Student), Wilfred Joseph Woods.

ACKNOWLEDGMENTS

The Council acknowledge with grateful thanks back numbers of the Journal from A. M. Desoutter, A.F.R.Ae.S.; S. Harrop, A.R.Ae.S.; W/Cmdr. A. K. Hunter, A.F.R.Ae.S.; Rotol Limited; Major B. W. Shilson, F.R.Ae.S.; E. Skelton, A.F.R.Ae.S.; and C. M. Strachan.

ADDITIONS TO THE LIBRARY

A.a.336—Introduction to Aerodynamics of a compressible fluid. H. W. Leipmann and A. E. Puckett. Wiley, New York. 1947.

BB.d.37—Ball and Roller Bearing Engineering. Arvid Palmgren. Skefko Ball Bearing Co. 1946.

C.c.64—Direction des Aérostats, deux modes d'orientation. Georges Racle. Auguste Chio, Paris. 1883.

E.d.33—Aircraft Drafting. Hyman H. Katz. Macmillan, New York. 1947.

EE.h.30—Thermodynamique des systèmes propulsifs a réaction et de la turbine a gaz. Maurice Roy. Dunod, Paris. 1947.

J.d.8—World Weather Record, 1931-1940. H. H. & F. L. Clayton. Smithsonian Institute. 1947.

L.d.116—*Notices to Airmen. Nos.* 259, 260, 261, 262, 265, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 283, 284, 285, 286, 287, 288.

M.a.32—Electronics and their application in industry and research. Bernard Lovell (Editor). Pilot Press. 1947.

R.c.291—Qualités que devront posséder les aéroplanes et les hélicoptères de l'avenir. Alfred Micciollo. Louis Vivien, Paris. 1909.

UB9/170—*Thornton Research Centre, Aero Engine Laboratory. Report No.* R170/47. *The Thornton C-P detonation limit gauge. P. G. Pigneguy.* 1947.

A.R.C. Reports and Memoranda

2144—*Mechanical instruments for solving linear simultaneous equations. R. A. Fairthorne.*

2016—*A theoretical analysis of the efficiency of sandwich construction under compressive end load. W. H. Wittrick.*

2156—*The effect of heat treatment on the Bauschinger effect on duralumin. C. Gurney and Z. Borysowski.*

2164—*Experiments on the effect of transition on control characteristics, with a note on the use of transition wires. W. W. Bryant and A. S. Batson.*

2162—*A theoretical discussion of wings with leading edge suction. M. J. Lighthill.*

2027—*A continuation of longitudinal stability and control analysis. Part I. General theory. S. B. Gates.*

2124—*Sandwich construction and core materials. Part II. The preparation of low density materials for use as cores in sandwich construction. N. E. Topp.*

2159—*Profile drag measurements on a Spitfire wing in flight at high speeds. W. A. Mair and W. J. Charnley.*

2141—*On some boundary value problems in the theory of the non-uniform supersonic motion of an aerofoil. Olga Todd.*

2173—*Piston cooling. H. R. Ricardo.*

2147—*The increase in the thrust obtainable in a power plant installation using the cooling-air as a propulsive jet. A. B. P. Beeton.*

2143—*The wrinkling of sandwich struts. F. T. Barwell and J. R. Riddell.*

2139—*Resistance derivatives of flutter theory. Part II. Results for supersonic speeds. A. R. Collar.*

2086—*Charts for the determination of the performance of a propeller under static, take-off, climbing and cruising conditions. A. B. Haines, A. R. C. McDougall and J. R. Monaghan.*

2175—*The buckling of a flat rectangular plate under axial compression and its behaviour after buckling. II. Conditions for permanent buckles. H. L. Cox.*

2066—*Tests of high-speed flow in right-angles pipe bends of rectangular cross-section. A. D. Young, G. L. Green and P. R. Owen.*

2065—*Pressure and wake measurements up to Mach number* 0.85 *on an EC*1250 *section with* 25 *per cent. control. J. A. Beavan, G. A. M. Hyde and R. G. Fowler.*

2161—*Minimum velocity aerofoils. L. G. Whitehead.*

2152—*High-speed wake traverse drag measurements on a model Spitfire half-wing. J. S. Thompson and W. Port.*

2174—*Two-dimensional supersonic channel design. Part I. A. O. L. Atkin.*

2132—*An analysis of static longitudinal stability in relation to trim and control force. S. B. Gates.*

2128—*Model experiments on the effect of wing engines on the natural frequencies and modes of vibration of wings. Part I. J. Hanson and E. Warlow-Davies. Part II. J. Taylor.*

2026—*Aerodynamic forces on wings in simple harmonic motion. W. Prichard Jones.*

2008—*The effect of boundary-layer thickness on the normal force distribution of aerofoils, with particular reference to control problems. A. S. Batson and J. H. Preston.*

2000 (*Monograph*)—*Mechanical admittances and their application to oscillation problems.*

N.A.C.A. Technical Memoranda

1151—*Lift and drag of wings with small span. F. Weinig.*

1163—*Drag corrections on high-speed wind tunnels. H. Ludwcig.*

1181—*Investigations on reduction of friction on wings, in particular by means of boundary layer suction. Werner Pfenninger.*

N.A.C.A. Technical Notes

1403—*Wind-tunnel investigation of the effect of tab balance on tab and control-surface characteristics. J. D. Brewer and M. J. Queijo.*

1400—*Hinge-moment characteristics of balanced elevator and rudder for a specific tail configuration on a fuselage in spinning attitudes. R. W. Stone and Sanger M. Burke.*

1399—*Preliminary investigation of a gas turbine with sillimanite ceramic rotor blades. F. J. Hartwig, B. W. Shefflin and R. J. Jones.*

1398—*Solutions for hydrodynamic impact force and response of a two-mass system with an application to an elastic airframe. W. L. Mayo.*

1396—*High-speed of an airfoil section cambered to have critical Mach numbers higher than those attainable with a uniform-load mean line. Donald J. Graham.*

1395—*Wind-tunnel investigation of the N.A.C.A. 654-421 airfoil section with a double slotted flap and boundary layer control by suction. J. H. Quinn.*

1393—*A flight investigation of the meteorological conditions conducive to the formation of ice on airplanes. William Lewis.*

1392—*Icing zones in a warm front system with general precipitation. William Lewis.*

1390—*Effect of compressibility on the distribution of pressures over a tapered wing of N.A.C.A. 230-series airfoil sections. E. O. Pearson, Jr.*

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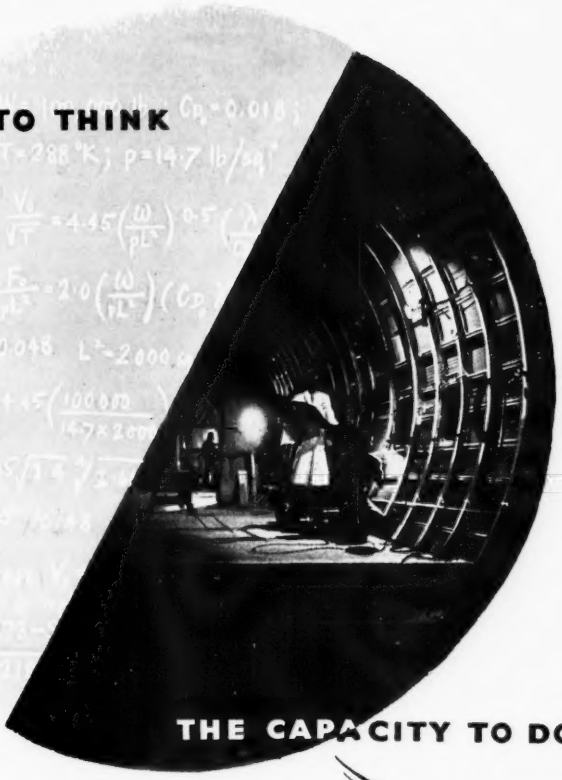

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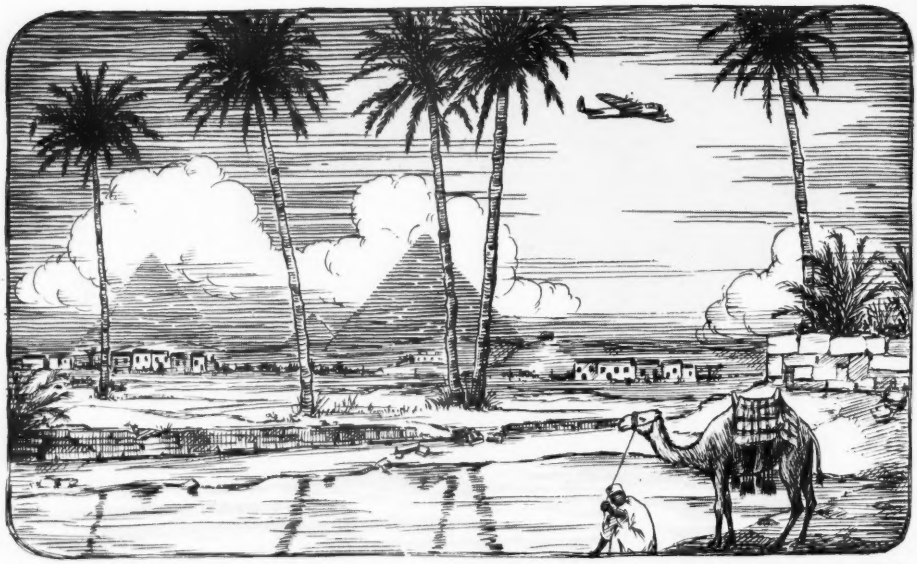

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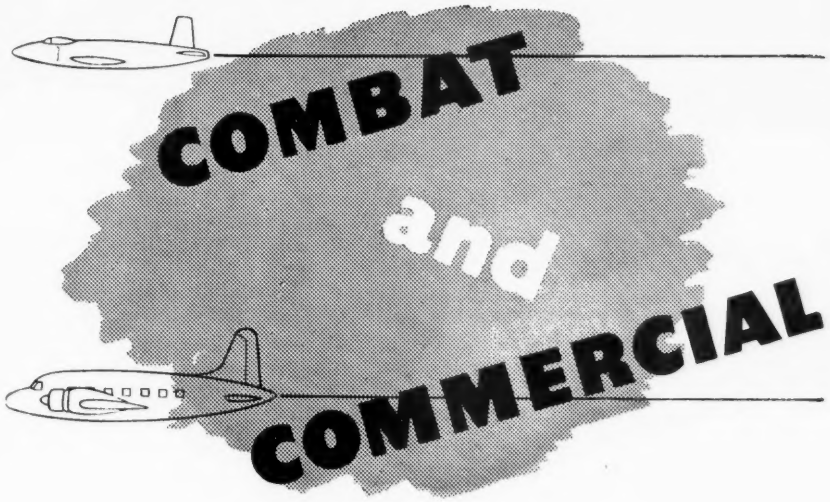

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AIRCRAFT SECTION

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The Journal is published monthly at the Offices of the Society, 4 Hamilton Place, Piccadilly, London, W1. None of the papers or paragraphs must be taken as expressing the opinion of the Council of The Royal Aeronautical Society unless such is definitely stated to be the case. Subscriptions per annum, £4 13 0 including postage; Single Numbers 7/6 or 7/9 post free. All communications for publication or Advertisements in the Journal, or on general matters affecting the Society should be addressed to—The Secretary, J. Laurence Pritchard, 4 Hamilton Place, W1. The Society's Bankers are Messrs. Coutts & Co.

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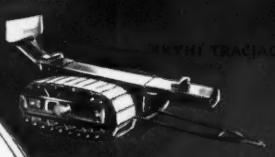

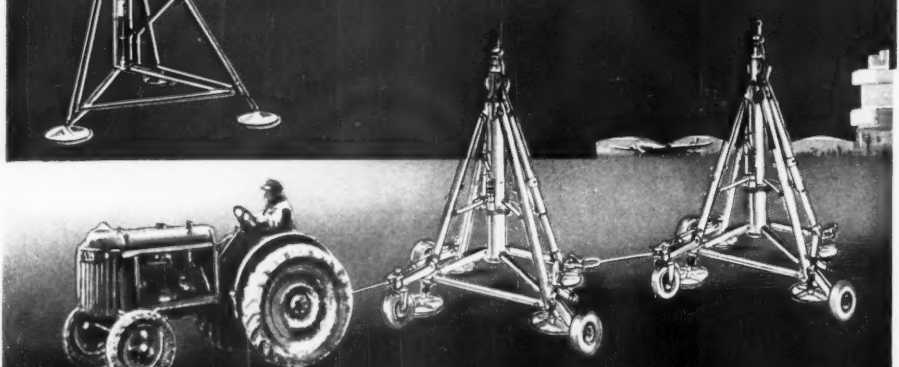

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THE ROYAL AERONAUTICAL SOCIETY

Paper Received February 1947

MINIMISING ZERO-DRIFT IN ELECTRICAL STRAIN GAUGE BRIDGES

by

C. R. URWIN, A.C.G.I., A.M.I.E.E.

and

K. H. SWAINGER, Ph.D., B.Sc.(Eng.)

Mr. Urwin was educated at Alleyne's School, Dulwich, London, became engineering apprentice with the Midland Railway Locomotive Department and then studied electrical engineering at the Imperial College of Science. After a period on the design staff of the British Electric Transformer Co. he joined the academic staff of the electrical engineering department of City and Guilds College, South Kensington, where he remained for a number of years and then joined the engineering staff of the International Standard Electric Corporation. Two years later he rejoined the staff of the City and Guilds College, specialising in Telecommunications and Electrical Measurements and became Senior Lecturer in Telecommunications.

Dr. Swainger was educated and served a mechanical engineering apprenticeship in Australia. Professional experience in general engineering followed in Australia and the Central Pacific. During the recent war he was at the Royal Aircraft Establishment, Farnborough, and initiated the laboratory for research into the causes of aircraft accidents, with emphasis on the structural aspects. He is at present I.C.I. Research Fellow in the University of London studying post-yield straining theoretically and experimentally.

SUMMARY

"Potential" leads are used to put strain gauge switches and leads in series with high resistance ratio arms in a modified Wheatstone Bridge. Thus the effects of switch contact resistance change and temperature-resistance change in leads can be minimised to any desired values. The high resistance ratio arms allow coarse resistance changes at the measuring bridge to measure the small resistance change at the gauge due to strain.

High potentials are applied to the bridge to minimise the effects of thermal e.m.f.

Active gauges and the temperature compensating gauges are in circuit for only a few seconds while the measurement is made.

1. INTRODUCTION

Electrical resistance wire strain gauges have been used in increasing number during the past five years. They have had the defect however that the zero or origin on the measuring scale drifts during the recording of increments of strain in a gauge. This drift can be caused by:—

(a) Unwanted and erratic change in the resistance value of the measuring arms of the electrical bridge.

The circuits proposed here were communicated to the Aeronautical Research Council, National Physical Laboratory, January 1946 and placed on Restricted Circulation. The data on gauge drift with time and current were noted and discussed with the A.R.C. in March 1946. Publication is now made by agreement with the Aeronautical Research Council.

(b) Variation of the total effective gauge resistance due to change of connecting lead resistance with temperature.

(c) Thermo-electric e.m.f.

(d) Variation of gauge resistance with time for a given current value.

(e) Variation in the structural arrangement of the gauge during the period of strain.

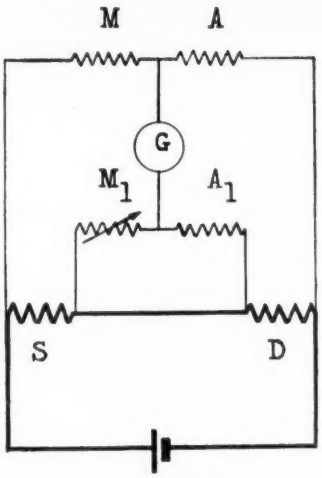
The gauges were for use in a materials research and it was required that the zero drift due to such changes as those indicated above should be as small as possible compared with the controlled incremental changes with strains applied. This report is concerned with the investigations made to minimise the defects mentioned in items (a), (b), (c) and (d). Item (e) will be discussed in another report after further investigation.

Although the bridges suggested here do not operate in the optimum electrical resistive condition, modern galvanometers are of sufficient sensitivity to offset any disadvantage due to loss of sensitivity caused by the circuits.

2. PRINCIPLES INVOLVED IN THE CIRCUIT DESIGN

2.1. High Resistance Ratio Arms

The Kelvin Double Bridge (Fig. 2.1) was first suggested as most suitable for the measurement of the small resistance changes in the wire gauges when strained. The high values of resistance in the ratio arms tend to minimise the effects of switch contact changes and temperature-resistance changes in the connecting leads from the gauges to the measuring bridge. It was seen that *the principle involved was to measure with "potential leads" the potential drop across the gauge due to the current supplied by "current leads"* (Fig. 2.2). Recognition of this principle led to the adoption of the more simple modified Wheatstone Bridge which does not involve electrically matched and mechanically coupled measuring resistances (Fig. 2.3). High value ratio arms of 5,000 to 10,000



Kelvin double bridge.
Fig. 2.1.

ohms are used successfully. Thus contact changes of gauge switches in series with such high resistance values have negligible effect. Similarly, the effects of changes of resistance in the " potential leads " are made negligible. Thus high valued ratio arms with potential leads disposes of defects (a) and (b) in para. 1.

Another advantage of very high ratio arms is that fairly coarse, cheap measuring boxes can be used. For example, with 5,500 ohms ratio arms a change of 1 ohm on the bridge corresponds to about 10^{-4} strain at the gauge.

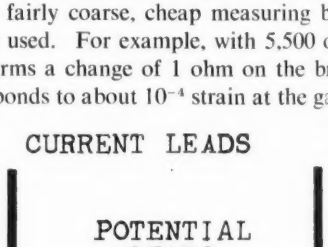
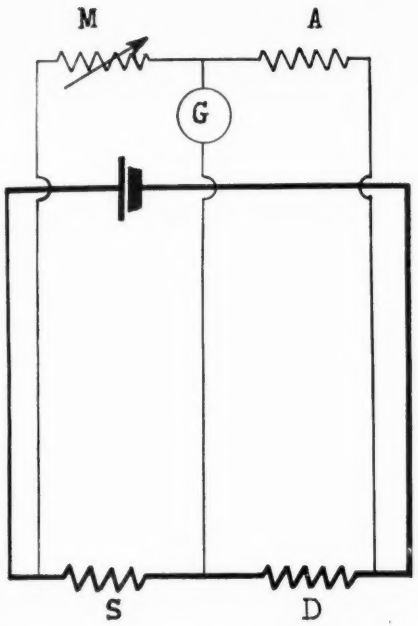


Fig. 2.2.



Wheatstone bridge modified with potential leads
Fig 2.3.

Galvanometer deflection is usually used to split the last ohm in measurement.

2.2. High potential applied to the measuring bridge.

The ordinary practice in Gt. Britain is to use very low potential on the bridge to keep gauge current values as low as possible (say 5 mA). Intuition appears to be the main reason for the practice and it was found on test that gauges operated quite satisfactorily with currents as high as 90 mA without fusing the wire filaments. The gauges were always cemented to metal specimens which undoubtedly cooled the gauges and prevented fusing of the filament.

Because of the many soldered joints the electrical circuits constructed under structural laboratory conditions are more prone to thermo-electric e.m.f. effects than those made by skilled electrical engineers. These thermal e.m.f. are in series with the high potentials applied to these bridges and this is the only known way to minimise their effects. Thus the use of high potential on the measuring bridge tends to minimise the effect of (c) in para. 1.

Another advantage of high potential on the bridge is that less sensitive galvanometers can be used than when the potential is low.

2.3. Gauges in circuit for short duration.

Tests gave the resistance variation of gauges with variation of current and time. The gauges tested were 200 ohms Nichrome 50 gauge wire in synthetic resin-impregnated paper as made by the British Thermostat Co. to National Physical Laboratory's specification. The construction left the winding core paper between the flattened helical grid or wire filament. Eight gauges were tested after cementing to an aluminium sheet with "Durofix" cellulose cement. A Kelvin Double

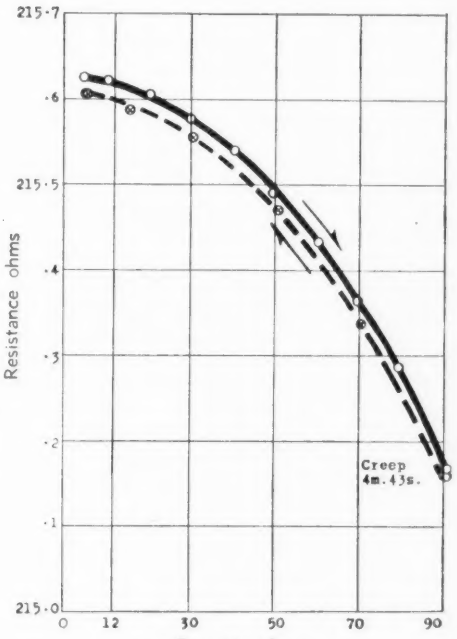


Fig. 2.4.

Bridge was used to measure the resistance values in a room of fairly steady temperature and humidity conditions.

Gauge resistance was found to decrease steadily with increasing current values. Fig. 2.4 shows a typical result. Current values were changed as rapidly as possible to keep the time effect to a minimum. A pause of 4 minutes 43 seconds occurred before decreasing the current from the highest value and the resulting "drifting" decrease by 0.010 ohm is of interest. This corresponds to a strain of 2.5×10^{-5}. Gauge resistance decreased steadily with time for a given current value. Fig. 2.5 shows a typical result. To check if the drift might be caused by the measuring bridge, a Tinsley Standard Resistance Box was substituted for the strain gauge after which no drift of value was observed over the 5 minutes' duration of the test. Fig. 2.6 shows a fairly typical set of current-time-resistance variations. There was no real consistency in this effect when different gauges were compared. At very low current values the resistance sometimes decreased, sometimes it did not change and once it increased slightly. In a middle range of current there *tended* to be more consistently little change of resistance with time. Higher currents gave changes in resistance of a more definite nature. These observations are brought out clearly in Fig. 2.7. The greatest and least variations in resistance within the group of gauges are plotted as the modulus of the difference after definite intervals of time from the switching on of current to the bridge. It appears that for these gauges at least there is a best working range of current between about 15 mA and 50 mA. The peaks of variation at low current values are of interest in view of the common practice at present. It should be borne in mind that the greatest and least values of which the difference is shown could be those for a given active gauge and compensating gauge in any given circuit under actual test conditions.

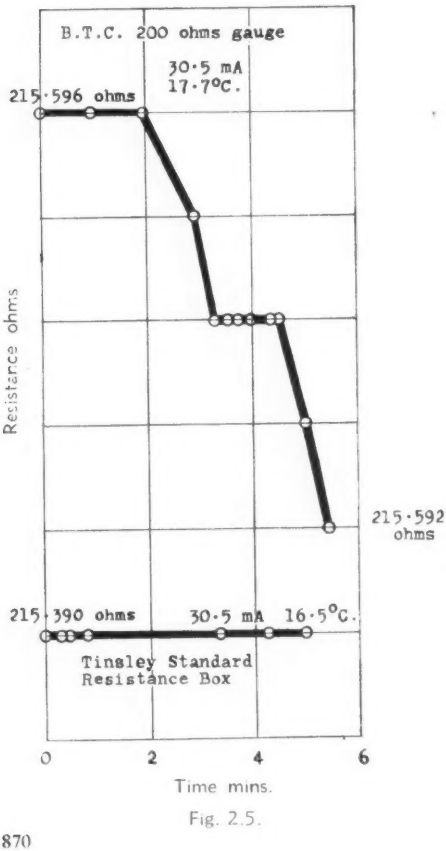


Fig. 2.5.

3. TEMPERATURE AND HUMIDITY CORRECTION ON THE STRAIN GAUGES

Suppose the bridge in Fig. 2.3 is balanced so that

$$A S = M D \quad . \quad . \quad . \quad (3.1)$$

Now allow the effective resistance of the strain gauge S and the "compensating" gauge D to change by the small values s_T and d_T respectively due to temperature and humidity variation. Again consider the balance condition

$$A(S + s_T) = M(D + d_T) \quad . \quad . \quad (3.2)$$

when M and A are not changed.

Mr. C. Y. Hsiung drew attention to an American reference stating that s_T and d_T were each linearly proportional to the initial

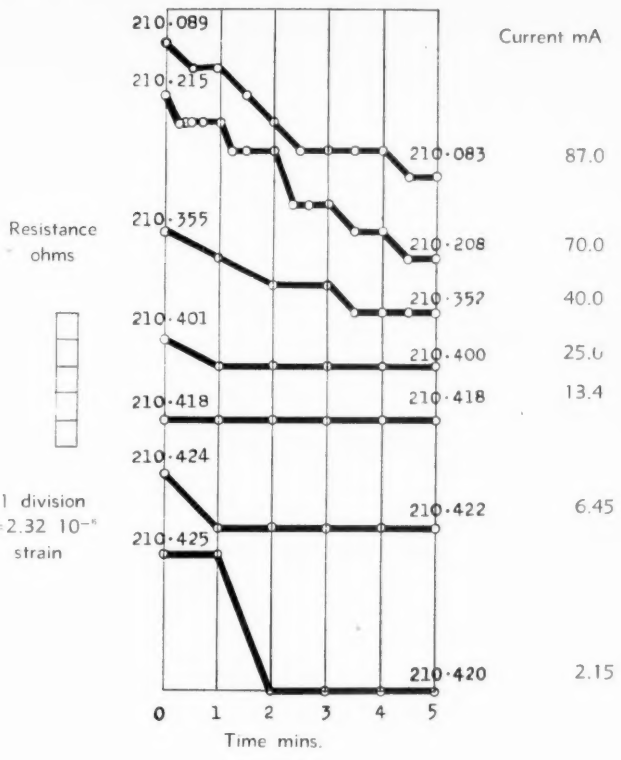


Fig. 2.6.

resistance of the gauges S and D respectively. Hence write

$$s_T = \phi S$$
$$d_T = \phi D \quad . \quad . \quad . \quad (3.3)$$

where ϕ is a function of temperature and humidity and of the geometry and construction of the gauge. It will have the same value in each gauge so long as they are geometrically and constructionally similar. Substitute (3.3) in (3.2) to give

$$AS(1+\phi) = M\,D(1+\phi) \quad . \quad (3.4)$$

which is evidently the condition (3.1) for balance. Hence there is no need to use a separate compensating gauge for each active gauge nor to match electrically D and S since (3.1) and (3.4) are not concerned with the absolute values of resistances. *Mechanical matching to give geometrical similarity appears to be more important than electrical matching.*

4. STRAIN GAUGE CALIBRATION

Because of strain let resistance S change to (S + s) and correspondingly M change to (M + m) to retain bridge balance (Fig. 2.3). Then the initial balance condition

$$A\,S = D\,M \quad . \quad . \quad . \quad (4.1)$$

and balance condition after strain together with (4.1) gives

$$A\,s = D\,m \quad . \quad . \quad . \quad (4.2)$$

As usual we calibrate another gauge S′ of similar construction on a member the strain of which is known and apply the calibration parameter to S on the understanding that geometrical similarity to S′ has been

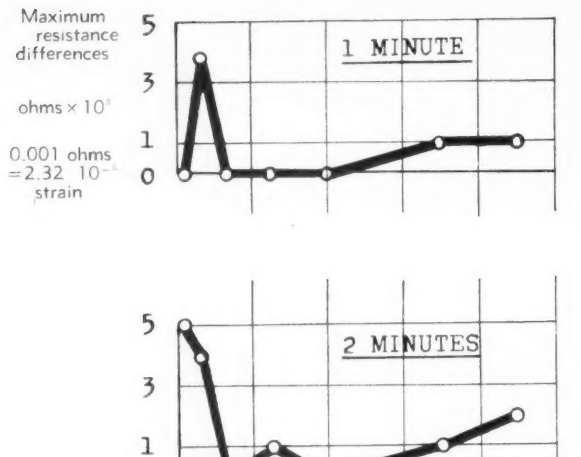


Fig. 2.7.

achieved. By definition calibration parameter is

$$c = (l/e)(s'/S') \quad . \quad . \quad (4.3)$$

where e is the strain of the member giving resistance change s′ in the gauge S′. Now using the geometrical similarity hypothesis

$$c = (l/e)(s/S) \quad . \quad . \quad . \quad (4.4)$$

Substitute (4.4) in (4.2) to give

$$A\,S = (Dm/ec)$$

and substituting (4.1) gives

$$e = (m/cM) \quad . \quad . \quad . \quad (4.5)$$

The strain e is linearly proportional to the change m made in the adjustable resistance with the initial balance value M.

5. INITIAL BALANCE OF THE BRIDGE

The practice of putting an "apex resistance" between M and A so that the galvanometer connection to the apex may be moved until balance is achieved appears undesirable. The calibration of the bridge is upset by adjusting the apex resistance and so changing the ratio M/A.

Consideration of (4.1) shows that if A and M follow the same law of resistance change with temperature and any changes are proportional to their initial resistance values then in para. 4 the balance is never upset. Hence it is proposed to adjust A to (A − a) to

account for initially unequal resistances in D and S.

6. MULTIPLE STRAIN GAUGE BRIDGES

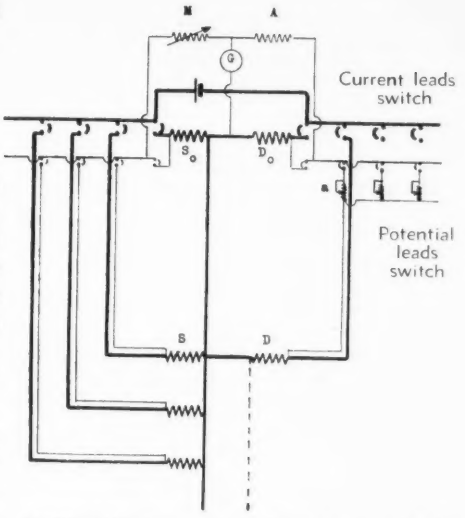
The experimental evidence and theory presented in the foregoing discussion led to the evolution of the circuit shown in Fig. 6.1.

Only one compensating gauge is used so long as the group of active gauges and the compensating gauge are more or less in the same ambient atmosphere as suggested in para. 3.

Zero adjusting resistance " a " is placed in series with one ratio arm A to suit para. 5.

A pair of resistances of about the same value as the gauges are built into the measuring bridge so that the bridge is in balance with continuous circuits when there is no active gauge switched into it. Active and compensating gauges are switched in only long enough to take a reading. This requires about 3 to 4 seconds. This complies with the evidence given in sub-para. 2.3.

If sub-groups of gauges operate under different temperature and humidity conditions then each sub-group must, of course, have its own compensating gauge. This is provided for by multiplying the switch points and connections on the compensating side of the bridge. Otherwise the circuit is the same as in Fig. 6.1. Four-layer medium quality wafer switches without special contacts are found suitable when potential leads and high ratio arms are used. Standard production gauges used in a structures laboratory require only one compensating gauge. No drift has been experienced during tests extending over the past twelve months.



Modified Wheatstone multiple strain gauge bridge
Fig. 6.1.

THE ROYAL AERONAUTICAL SOCIETY

Paper received March 1947

THE INFLUENCE OF AIRCRAFT GROSS WEIGHT UPON THE SIZE AND WEIGHT OF HULLS AND FUSELAGES

by

L. W. ROSENTHAL, A.F.R.Ae.S.

After serving an apprenticeship in general and marine engineering Mr. Rosenthal entered the Drawing Office of Saunders-Roe Ltd. in 1935. *He transferred to the Weights Section in* 1936 *and has been responsible for its work for the past six years. He is an Associate Fellow of the Society and an Associate Fellow of the American Society of Aeronautical Weight Engineers.*

INTRODUCTION

In this paper an attempt has been made to trace the influence of gross weight, as the accepted measure of size, upon the dimensions of hulls and fuselages and from this, to evaluate the probable trend of the weights of these components with increasing gross weight.

During the past few years the size and gross weight of aircraft have rapidly increased and there appears at the moment little, if any, structural limit upon the size and gross weights attainable, since it can be shown that the percentage weights of hulls and fuselages tend to decrease as size increases.

This paper should not be interpreted as an attempt to predict the actual body dimensions and weights of future large aircraft. Its purpose is to use the data relating to aircraft so far built and projected, and from them to indicate the trend to be expected on future large designs. The upper limits of the curves extend into the sphere of gross weights of 300,000 lb. to 500,000 lb., and the information is presented with some realisation of the risk taken in attempting to look into the future.

The geometrical data have been gathered from the normal trade sources and journals and the weight data are based upon a comprehensive and up-to-date survey of modern types.

METHOD

In an endeavour to establish some relationship between size and gross weight, hull and fuselage dimensions have been plotted against the cube root of the gross weight in Figs. 1 and 2, which for purposes of comparison, are combined in Fig. 3.

To avoid confusion, only the " full depth " hulls have been plotted on the hull depth curve, since the inclusion of parasol types (*e.g.* Catalina) would have been misleading. For similar reasons these types have again been avoided when plotting hull surface area.

In plotting hull beam, a beam loading of 1.0 has been taken as approximating closely to present day practice, although a curve might have been drawn through the plots, rather than the straight line indicated. The question of beam variation on future large aircraft is touched upon later.

Only fuselages of a size suitable for transport purposes have been used, and these have been broken into two groups: single and double deck, the former being mainly cylindrical in shape, and the latter being the now

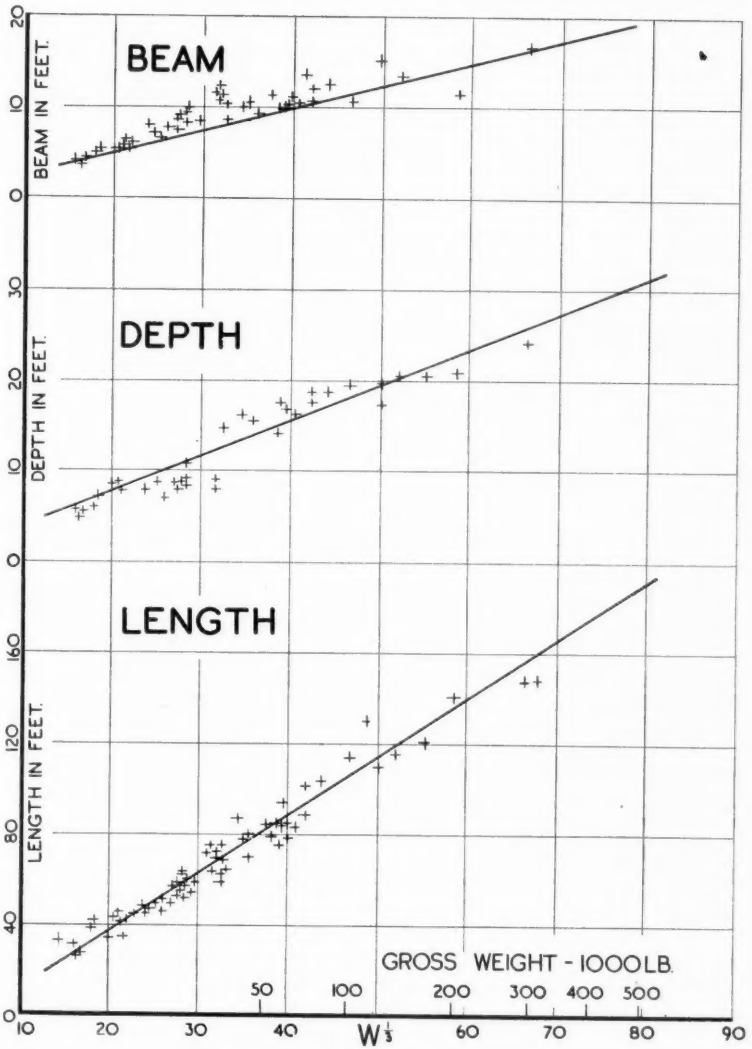
familiar "double-bubble" type of the Curtiss Commando and the Boeing Stratocruiser.

The interpretation of gross weight used throughout this analysis is taken as the maximum gross weight generally associated with a particular type, and corresponds to a flight factor of the order of 4.

The interpretation of hull and fuselage weight is taken as the weight for the complete component, including all the items detailed upon Page 7 of M.O.S. Weight Summary Form 2492.

DISCUSSION

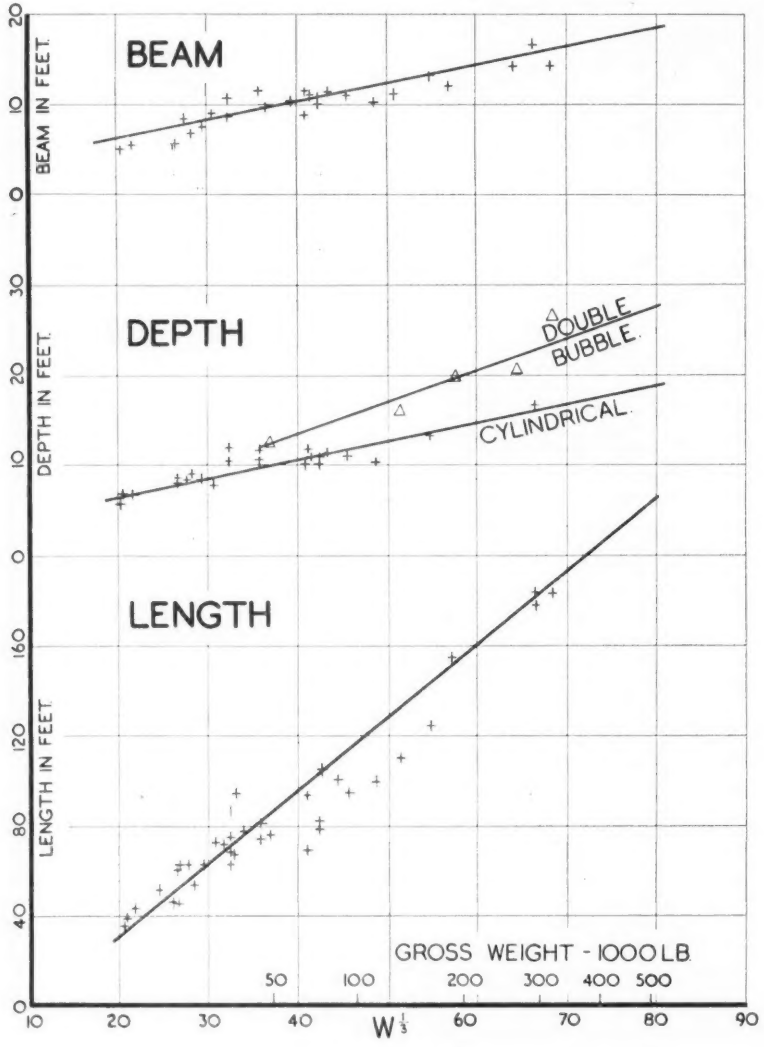
Consideration of the curves confirms the fact that over the range of data available,



Hull dimensions.
Fig. 1.

dimensions are in general directly proportional to the cube root of the gross weight (*i.e.* the scale) and that there is no evidence, at the moment, to suggest that aircraft bodies are getting relatively smaller as all-up weights increase. In this connection, however, when more data becomes available, an investigation into the effect of range upon body size at a given gross weight might be worth while, since one would expect the body of the very long range aircraft to be relatively smaller, since a larger proportion of the disposable load would be carried in the wing.

As shown in Fig. 7 the percentage weights of aircraft bodies tend to decrease slightly with size. This is important, because it clears

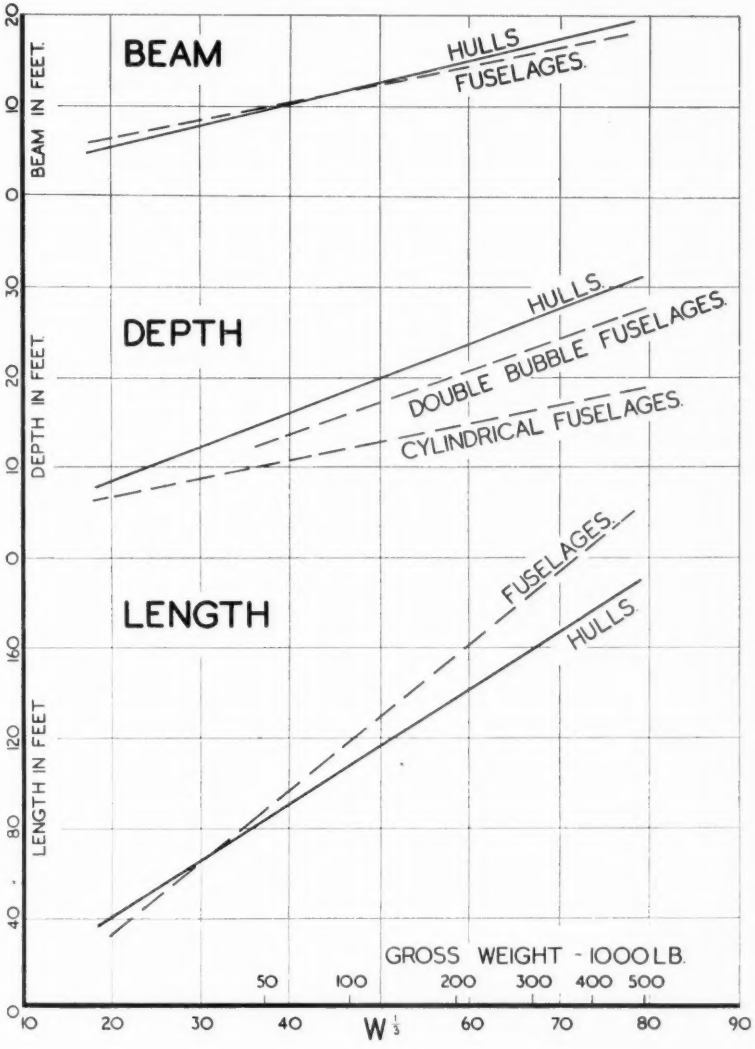


Fuselage dimensions.
Fig. 2.

away any doubts one may have had about the effects of the square cube law making large structures impossible.

DIMENSIONS

The beams of hulls and fuselages are approximately the same at the same gross weights; the hulls however are deeper in all sizes where some similarity between them and the "double-bubble" fuselages might have been expected. In this connection it is known that on one large passenger-carrying flying-boat, accommodation considerations (mainly head room) just aft of the main step, together with the aft body angle, influenced the depth of hull below the floor on the fore-



Comparison of dimensions.
Fig. 3.

body. From this, it seems reasonable to anticipate conditions of a similar nature on larger aircraft, leading again to a hull deeper than the corresponding double-deck fuselage.

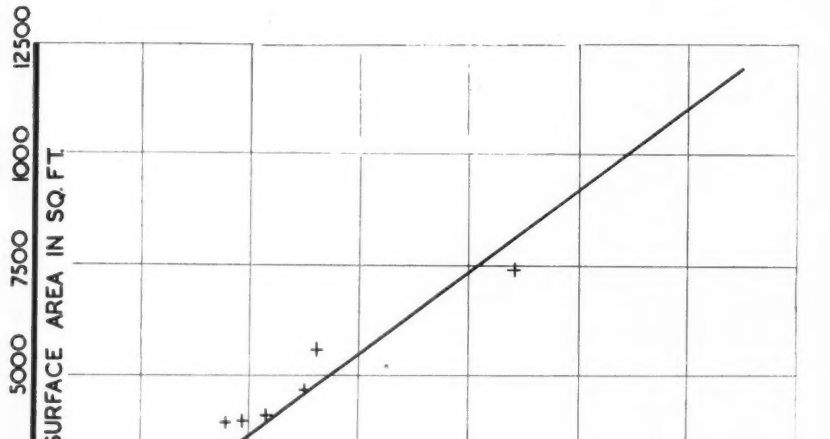
In view of the accommodation problem, it might be expected that body depth would increase in definite steps as additional decks become necessary. This effect cannot be clearly traced on the curves of hull and fuselage depth (see Figs. 1 and 2), although it is true that in the range of British civil landplanes at present being produced, fuselage depth remains reasonably constant at gross weights between 40,000 lb. and 90,000 lb. While keeping this latter point in mind it is felt that its effect is sufficiently small to be ignored in a paper of such a general nature as this. It will result in the fuselage weights for this group of aircraft being below the curve in Fig. 7.

Fuselages tend to be longer than hulls at all gross weights above about 40,000 lb. This greater length, combined with the smaller vertical dimension, should lead to a smaller and therefore lighter tail unit on the landplane, although at the moment tail unit weights and the various tail layouts do not lend themselves to satisfactory analysis. The abnormal proportion of fuselage forward of the wing on some recent types will probably cancel any advantage the landplane may have gained.

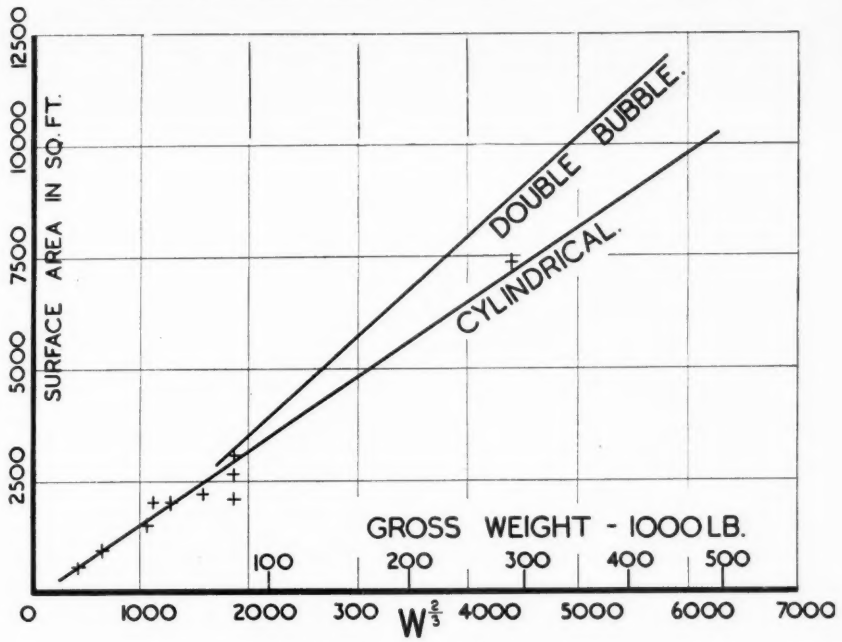
Some tendency to break away from the linear relationship is apparent upon the fuselage length curve (Fig. 2) between 60,000 lb. and 125,000 lb., but the plots re-establish themselves again at gross weights over 200,000 lb.

SURFACE AREA

Since the dimensions appear proportional to the cube root of the gross weight, it is reasonable to plot surface area against the



Surface area—hulls.
Fig. 4.



Surface area—fuselages.
Fig. 5.

two-thirds root of the weight. This has been done in Figs. 4 and 5.

The curve for the area of cylindrical fuselages is based upon British aircraft and it is of interest to note that the latest American cylindrical designs fall consistently below this curve.

The corresponding curve for the "double-bubble" fuselages is presented as a reasonable estimate of the surface area for this type, although the foundations are admittedly slender. It is based upon the scanty data available on American types, since no British aircraft with this fuselage shape have so far appeared.

On any particular design the most potent way of reducing body weight is by reducing surface area. This leads to the notion of "tailoring" the fuselage for the particular function for which the aircraft is intended. Density of load to be carried will exert its influence here and the freighter aircraft should have a smaller body than the passenger-carrying aircraft for the same range and gross weight. An interesting sidelight upon this might be shown by an analysis of the operating economics of, say, a Lancastrian and York used respectively for freight and passenger-carrying over the same route.

Another factor influencing surface area is the fuselage fineness ratio, *i.e.* length divided by depth. For the same volume, the lower the fineness ratio the smaller the surface area. For passenger work the whole problem is confused by the difficulty of establishing a desirable volume and floor area per passenger, and since this is tied up with time in the air, *i.e.* duration of trip, the problem of finding the most economic fuselage geometry might well appear insoluble. Generally, it can be said that the smaller the fineness ratio the smaller the surface area for a given volume and, at the expense of repetition, the smaller the area, the lower the body weight.

The existing "double-bubble" fuselage types, with the possible exception of the Boe-

ing Stratocruiser, appear to have been designed for high density medium-range work, since their surface areas, and presumably volumes, are greater than those of the corresponding cylindrical types.

SPECIFIC WEIGHTS

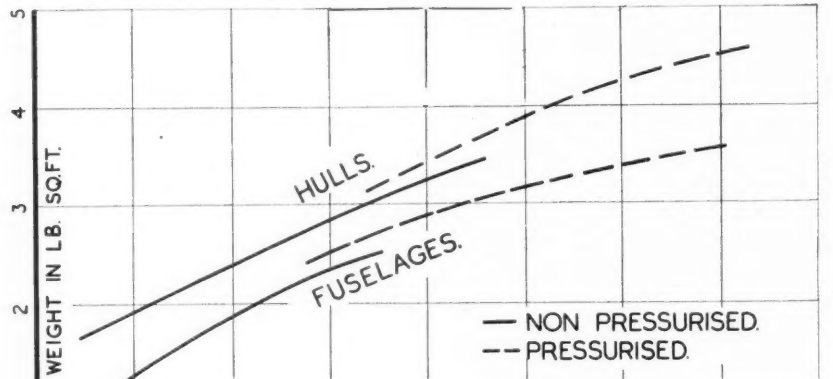
The specific weights are plotted in Fig. 6 solely on a statistical basis and the fact that the hull weights plot almost as a straight line against the cube root of the gross weight, appears purely fortuitous. This is to some extent borne out by the fact that the corresponding fuselage curves appear to follow a slightly different law.

A simple theoretical approach on the matter of specific weights suggests that, at least for unpressurised components, the curves should be straight when plotted against the two-thirds root of the gross weight. Divergence from this theoretical curve will be conditioned by improvements in design technique, improvements in material and, possibly on recent aircraft, by modification of the stressing conditions.

It may be argued that the total body weights, *i.e.* including flooring, windows and so on, are not the correct weights upon which to establish overall specific weight curves. Against this argument, it is submitted that this paper does not set out to do more than trace the general effect of gross weight upon body size and weight; and further, that the data necessary to establish a more accurate approach will be difficult to collect, since component weights are more easily obtained than detailed breakdowns.

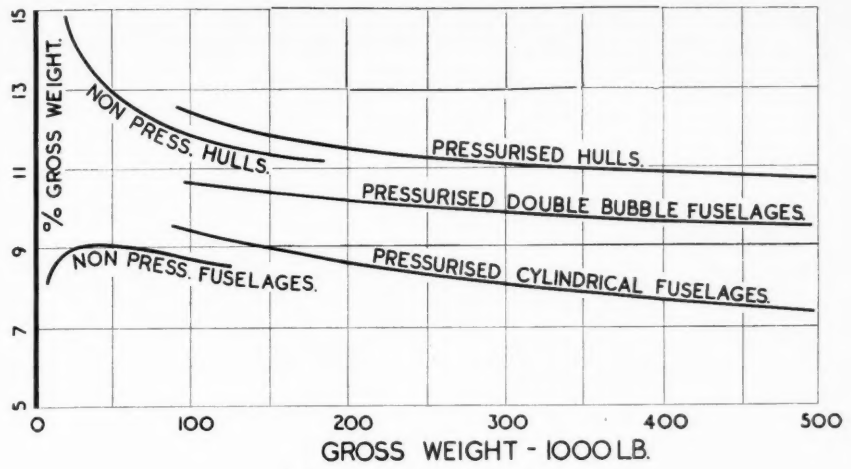
In spite of this the specific weight curves presented, used with actual surface areas, will be found to give satisfactory weight estimates for new projects.

A curve for the specific weight of pressurised fuselages is given. This curve, possibly the first published on pressurised types, is based upon upwards of nine aircraft and can be taken as giving a good indication of the structural penalty involved. This represents a weight increase of approximately 8 per cent. on non-pressurised fuselages built in this country.



Specific weights

Fig. 6.



Trend of body weight with size.
Fig. 7.

A curve for pressurised hulls is suggested; there are indications that despite the fact that the normal hull shape is far from ideal for pressurising, the penalty on a hull may be slightly less than the corresponding fuselage figure, because the inherently heavier bottom structure of the hull may be economically adapted to carry some of the pressurising loads. The curve given is some 6 per cent. above the weight expected for unpressurised hulls.

It is not possible at the present state of knowledge to say how much extra structure weight will be necessary when cabin pressures exceed the present 5½ lb. to 8 lb. per square inch. It may be that when knowledge of the problem becomes greater we may see some decrease in specific weight, even though pressures may rise.

COMPONENT WEIGHTS

In Fig. 7 the trend of component weight with size is plotted. These curves are calculated from the specific weights and areas given in Figs. 4, 5 and 6.

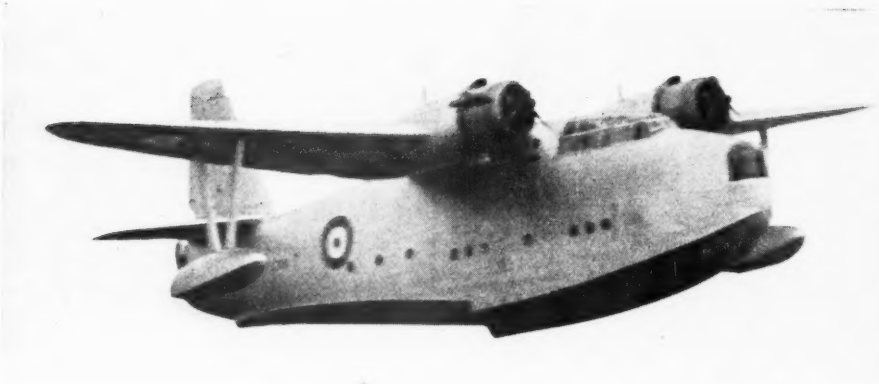
It is interesting to plot the points for actual hulls and British fuselages upon these curves, where, in general, surprisingly good agreement will be found. American fuselage weights will tend to be lower than the curves, mainly because of their smaller surface area, to which reference is made in an earlier paragraph.

HULL WEIGHTS

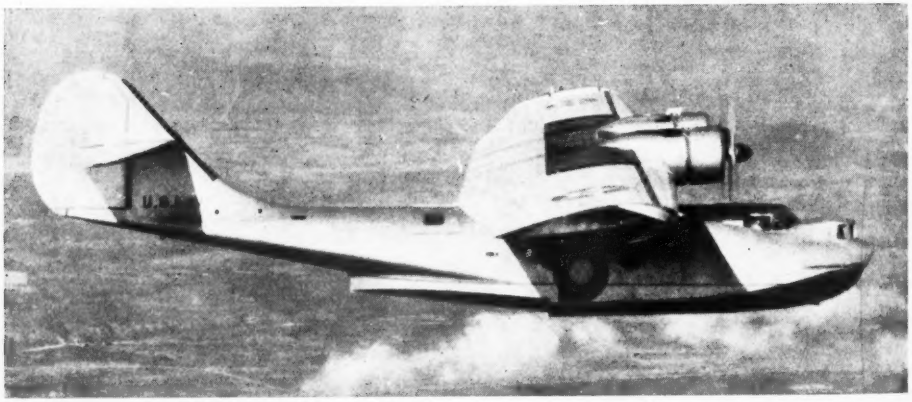
Comparison between the hull weight curve and actual weights is particularly encouraging. It should be noted that the curve as drawn is based mainly upon hulls having fore-body length-to-beam ratios of less than four. There are indications that for hydrodynamic reasons there may be a tendency to increase this ratio to higher values, and although this would probably be accompanied by an increase in beam loading, the overall effect would be to increase the total surface area of the hull.

The decrease in beam would effect a considerable saving on the bottom frame weight, but the increase in surface area would no doubt absorb all of this, and the final result might be some increase in hull weight.

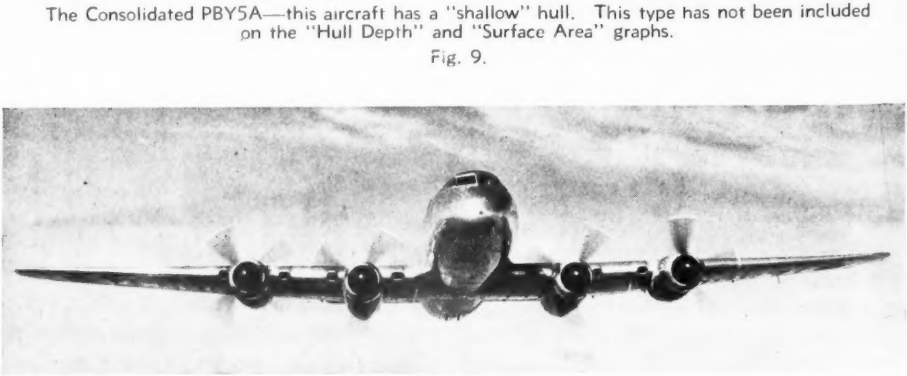
That this potential weight increase can be " designed " out by careful attention to detail and geometry, is borne out by the weight of the hull of the Blöhm and Voss 222 (approx.



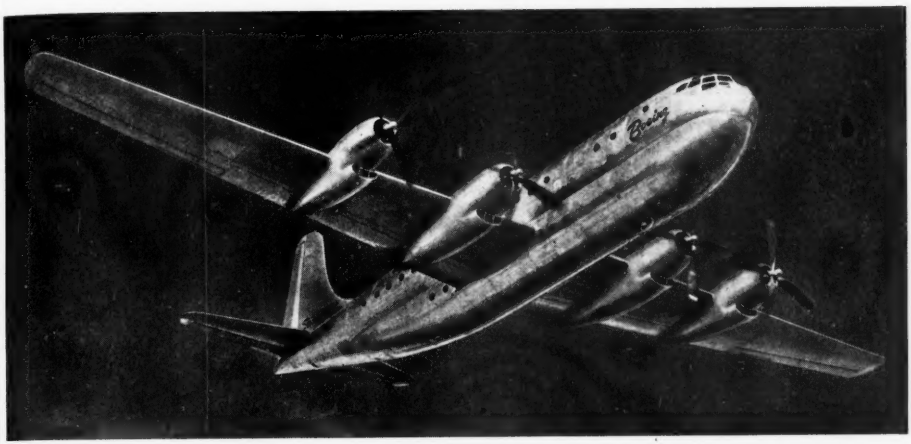
The Saunders-Roe Lerwick—a typical "full depth" hull.
Fig. 8.



The Consolidated PBY5A—this aircraft has a "shallow" hull. This type has not been included on the "Hull Depth" and "Surface Area" graphs.
Fig. 9.



The Handley Page Hermes—a modern example of the cylindrical type of fuselage.
Fig. 10.



The Boeing Stratocruiser—a good example of the double-bubble type of fuselage.
Fig. 11.

12 per cent.), which had a fore-body length-to-beam ratio of 4.4. It does appear however, that from a weight aspect, any tendency to increase hull length should be strongly resisted until it is proved that satisfactory hydrodynamic performance cannot be maintained with the orthodox geometry.

Fuselage Weights

The general arguments put forward about surface area and fineness ratio need not be repeated here, except to note that lower fineness ratios will produce deeper fuselages. Fuselage depth does not appear to exert any clearly defined influence upon specific weight, but there are vague indications that it should not be ignored, since in one or two cases the deep fuselages plot on the lower side of the specific weight curves in Fig. 6.

The influence of the "gust cases" upon the weight of slender fuselages will soon be known and it may be found that these also will become an important factor in the selection of fuselage shape.

FURTHER DEVELOPMENTS

To improve upon the existing weights of hulls and fuselages, further investigations into the various relevant factors will be necessary and the following points are suggested as lines which might well be studied:

1. A detailed investigation into the body structure weights, treating the non-structural items separately.
2. An investigation into standards and practice regarding passenger comfort, to find desirable volume and floor area figures.
3. The effect of range upon body size and detailed investigation into the best fuselage shape.
4. The effect of gust, tail and landing loads, in relation to body depth.

THE ROYAL AERONAUTICAL SOCIETY

Paper submitted May 1947.

THE GENERAL THEORY OF CYLINDRICAL AND CONICAL TUBES UNDER TORSION AND BENDING LOADS

Single and Many Cell Tubes of Arbitrary Cross-Section with Rigid Diaphragms

PART V—*Concluded*

by

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Parts I to IV of this paper were published in the February 1947 *Journal and the first half of Part V in the September* 1947 *Journal.*

5.4.13. *Alternative procedures for determining* ξ_i, η_i.

In 3.2.1 it was indicated that ξ_i, η_i may be found from the appropriate co-factors of the secular equation and appear directly as functions of the geometric and elastic properties of the root cross-section. The method will be applied in 5.6.2 for the four-boom tube, in 5.7.3 for the n-boom tube, and in 5.8 for the trapezoidal tube with direct stress carrying top and bottom panels. This procedure is advantageous when it is desired to study the effect on ξ, η of varying particular parameters of the cross-section.

However, when in any case the h-functions have to be obtained, it is often more expedient to determine ξ, η by one of the following procedures.

(*a*) *For tubes with direct stress carrying walls.*

In such cases, having first obtained the relative magnitudes of the C-coefficients, one substitutes expression (56) for the h_i-functions in the three equilibrium conditions (54) and thus one obtains three linear simultaneous equations in,

$$\int_S \frac{h_1}{t}\,ds,\ \xi_i \oint_S \frac{h_1}{t}\,ds,\ \text{and}\ \eta_i \oint_S \frac{h_1}{t}\,ds$$

whence ξ_i, η_i are determined.

(*b*) *The n-boom tube with walls effective only in shear.*

Only the single-cell tube will be considered. Multi-cell tubes of the type shown in Fig. 16 are reduced to an equivalent single-cell tube with the same ξ_i, η_i values by the method of 4.3.2 as generalised in 5.7.4.

In this case the h_i-functions are determinable as to magnitude without first finding ξ_i, η_i. The complete analysis will be given in 5.7.1-3. Having the h_i-functions it is then possible to substitute their values in any two of equations (66a) and to solve for ξ_i, η_i. However, a more symmetrical procedure is as follows:

Multiplying equations (66a) by,

$$\frac{t_{m,\,m+1}}{s_{m,\,m+1}}(x_{m+1}-x_m)$$

summing over all walls and using the last of relations (67) one obtains,

$$\sum_1^n \frac{\Delta h_m}{B_m}\left\{\frac{x_{m-1,\,m}}{\delta_{m-1,\,m}}-\frac{x_{m,\,m+1}}{\delta_{m,\,m+1}}\right\}$$
$$=\frac{\lambda^2}{2A}\left\{2\sum_1^n A_{m,\,m+1}\frac{x_{m,\,m+1}}{\delta_{m,\,m+1}}-\xi\sum_1^n\frac{(x_{m,\,m+1})(y_{m,\,m+1})}{\delta_{m,\,m+1}}+\eta\sum_1^n\frac{(x_{m,\,m+1})^2}{\delta_{m,\,m+1}}\right\}\sum_1^n h_{m,\,m+1}\delta_{m,\,m+1} \quad . \quad (342a)$$

and similarly multiplying equation (66a) by,

$$\frac{t_{m,\,m+1}}{s_{m,\,m+1}}(y_{m+1}-y_m)$$
$$\sum_1^n \frac{\Delta h_m}{B_m}\left\{\frac{y_{m-1,\,m}}{\delta_{m-1,\,m}}-\frac{y_{m,\,m+1}}{\delta_{m,\,m+1}}\right\}$$
$$=\frac{\lambda^2}{2A}\left\{2\sum_1^n A_{m,\,m+1}\frac{y_{m,\,m+1}}{\delta_{m,\,m+1}}-\xi\sum_1^n\frac{(y_{m,\,m+1})^2}{\delta_{m,\,m+1}}+\eta\sum_1^n\frac{(x_{m,\,m+1})(y_{m,\,m+1})}{\delta_{m,\,m+1}}\right\}\sum_1^n h_{m,\,m+1}\delta_{m,\,m+1} \quad . \quad (342b)$$

where,

$$\delta_{m,\,m+1}=\frac{s_{m,\,m+1}}{t_{m,\,m+1}} \quad . \quad (343)$$

$$\Delta h_m = h_{m,\,m+1}-h_{m-1,\,m} \quad . \quad (344)$$

$$\left.\begin{aligned}x_{m,\,m+1}&=x_{m+1}-x_m\\ y_{m,\,m+1}&=y_{m+1}-y_m\end{aligned}\right\} \quad . \quad (345)$$

The simultaneous equations (342) may be solved at once for ξ and η.

If the section is symmetrical about $G\overline{Y}$

$$\sum_1^n\frac{(x_{m,\,m+1})(y_{m,\,m+1})}{\delta_{m,\,m+1}}=0 \quad . \quad (346)$$

$$\sum_1^n A_{m\,m+1}\frac{y_{m,\,m+1}}{\delta_{m,\,m+1}}=0 \quad . \quad (347)$$

and the flexural modes in the $G\overline{Y}$ direction have to be considered separately from the remaining modes. For the $G\overline{Y}$ flexural modes condition (335) is satisfied, *i.e.*,

$$\sum_1^n h_{m,\,m+1}\delta_{m,\,m+1}=0 \quad . \quad (335a)$$

and ξ is infinite. However, from equation (342b)

$$a_x=\xi\sum_1^n h_{m,\,m+1}\delta_{m,\,m+1}=-\sum_1^n\frac{\Delta h_m}{B_m}\left\{\frac{y_{m-1,\,m}}{\delta_{m-1,\,m}}-\frac{y_{m,\,m+1}}{\delta_{m,\,m+1}}\right\}\Big/\frac{\lambda^2}{2A}\sum_1^n\frac{(y_{m,\,m+1})^2}{\delta_{m,\,m+1}} \quad . \quad (348)$$

η is of course finite but indeterminate, as can be seen from equation (342a).

In the remaining modes by symmetry $\xi=0$ and from equation (342a),

$$\eta\sum_1^n h_{m,\,m+1}\delta_{m,\,m+1}=\sum_1^n\frac{\Delta h_m}{B_m}\left\{\frac{x_{m-1,\,m}}{\delta_{m-1,\,m}}-\frac{x_{m,\,m+1}}{\delta_{m,\,m+1}}\right\}\Big/\frac{\lambda^2}{2A}\sum_1^n\frac{(x_{m,\,m+1})^2}{\delta_{m,\,m+1}} \quad . \quad (349)$$

η will in general be finite, but in doubly symmetrical cross-sections there will exist a set of flexural modes in the $G\overline{X}$ direction for which condition (335a) is again satisfied with η infinite; the product on the L.H.S. of equation (349) then becomes a_y which has a definite value.

For cross-sections with no axis of symmetry it is always possible to find a system of axes which satisfy the following conditions:

$$\sum_{1}^{n} \frac{(x_{m,\,m+1})\,(y_{m,\,m+1})}{\delta_{m,\,m+1}} = 0 \qquad . \quad . \quad . \quad . \quad . \quad (346)$$

which defines the direction of the axes, and

$$\sum_{1}^{n} A_{m,\,m+1} \frac{y_{m,\,m+1}}{\delta_{m,\,m+1}} = \sum_{1}^{n} A_{m,\,m+1} \frac{x_{m,\,m+1}}{\delta_{m,\,m+1}} = 0 \qquad . \quad . \quad . \quad . \quad . \quad (350)$$

which define the origin.

For this special system of axes the values ξ_i, η_i for all modes are again given by equations (348) and (349).

It is interesting at this point to consider the physical meaning of the axes defined by equations (346) and (350) in connection with equation (21) of 2.3. In fact conditions (346) and (350) may be rewritten,

$$\oint t' \sin\psi \cos\psi ds' = 0 \qquad . \quad . \quad . \quad . \quad . \quad (346a)$$

and

$$\oint t'p' \sin\psi ds' = \oint t'p' \cos\psi ds' = 0 \qquad . \quad . \quad . \quad . \quad (350a)$$

so that equations (21) become,

$$\left.\begin{aligned} \frac{du}{dz'} \oint t' \sin^2\psi ds' &= \frac{Q_x}{G} \\ \frac{dv}{dz'} \oint t' \cos^2\psi ds' &= \frac{Q_y}{G} \\ \frac{d\theta}{dz'} \oint t'p'^2 ds' &= \frac{T}{G} \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (21a)$$

Thus if a tube is loaded so that there is no warping the *shear* deflections due to Q_x and Q_y occur only in the directions X and Y respectively of the axes defined by equations (346a) and (350a). In a single-cell tube under pure torque the warping is zero if

$$tp = \text{constant (see 2.2.)}$$

and with this condition the last of equations (21a) reduces to the Bredt-Batho formula for rate of twist. A three-boom tube with walls effective only in shear has zero warping under any loading so that equations (21a) for the *shear* deflections and the twist are then true under any loading.

5.4.14. *The deflections and twist of the tube under arbitrary loading.*

Apart from the warping, the deformation of the tube is completely described by the twist θ of the diaphragms and the deflections u_E and v_E of the line OE about which the torque T_{BE} is referred.

The displacements u, v of the line OZ are related to u_E, v_E by the equations,

$$\left.\begin{aligned} u_E &= u - \rho\eta_E\theta \\ v_E &= v + \rho\xi_E\theta \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (351)$$

The twist θ can best be obtained from equation (35) of 3.2 with $t' = t^* t$

$$\frac{d\theta}{dr} = -\frac{1}{2AG\rho t^*} \oint_S \frac{1}{t} \frac{\partial F}{\partial r} ds \qquad . \quad . \quad . \quad . \quad . \quad (35a)$$

From equation (295), using equations (268) and (277),

$$\frac{\partial F}{\partial r} = \sum_i \frac{dg_i}{dr} h_i - \frac{T_{BE}}{2A\rho^2} h_B + \frac{Q_x}{\rho S} h_X + \frac{Q_y}{\rho S} h_Y$$

Thus equation (35a) becomes,

$$\frac{d\theta}{dr} = \frac{1}{4A^2G}\oint_S \frac{h_B}{t}\,ds \cdot \frac{T_{BE}}{\rho^3 t^*} - \frac{1}{2AGS}\oint_S \frac{h_X}{t}\,ds \cdot \frac{Q_x}{\rho^2 t^*} - \frac{1}{2AGS}\oint_S \frac{h_Y}{t}\,ds \cdot \frac{Q_y}{\rho^2 t^*}$$
$$- \frac{1}{2AG}\sum_i \left\{\oint_S \frac{h_i}{t}\,ds \cdot \frac{1}{\rho t^*}\frac{dg_i}{dr}\right\} \quad . \quad . \quad . \quad . \quad . \quad (352)$$

If E is the shear centre,

$$\oint_S \frac{h_X}{t}\,ds = \oint_S \frac{h_Y}{t}\,ds = 0 \text{ (see 5.4.3)}$$

and equation (352) reduces to,

$$\frac{d\theta}{dr} = \frac{1}{4A^2G}\oint_S \frac{h_B}{t}\,ds \cdot \frac{T_{BE}}{\rho^3 t^*} - \frac{1}{2AG}\sum_i \left\{\oint_S \frac{h_i}{t}\,ds \cdot \frac{1}{\rho t^*}\frac{dg_i}{dr}\right\} \quad . \quad . \quad . \quad (352a)$$

Note that from equation (315) the last terms on the R.H.S. of equation (352) and (352a) may also be written,

$$+ \frac{1}{4A^2G}\sum_i \left\{T_{Bio}\left[\oint_S \frac{h_i}{t}\,ds\right]^2 \frac{G_{ri}}{\rho}\right\} \quad . \quad . \quad . \quad . \quad (352b)$$

Integrating equation (352) with the condition $\theta = 0$ at $r = r_o$ one obtains,

$$\theta = -\frac{1}{4A^2G}\oint_S \frac{h_B}{t}\,ds \cdot \int_r^{r_o} \frac{T_{BE}}{\rho^3 t^*}\,dr$$
$$+ \frac{1}{2AGS}\oint_S \frac{h_X}{t}\,ds \cdot \int_r^{r_o} \frac{Q_x}{\rho^2 t^*}\,dr + \frac{1}{2AGS}\oint_S \frac{h_Y}{t}\,ds \cdot \int_r^{r_o} \frac{Q_y}{\rho^2 t^*}\,dr$$
$$- \frac{1}{4A^2G}\sum_i \left\{T_{Bio}\left[\oint_S \frac{h_i}{t}\,ds\right]^2 \int_r^{r_o} \frac{G_{ri}}{\rho}\,dr\right\} . \quad . \quad . \quad . \quad . \quad . \quad (353)$$

Similarly for the cylindrical tube,

$$\theta = -\frac{1}{4A^2G}\oint_S \frac{h_B}{t}\,ds \cdot \int_{z'}^{c} \frac{T_{BE}}{t^*}\,dz'$$
$$+ \frac{1}{2AGS}\oint_S \frac{h_X}{t}\,ds \cdot \int_{z'}^{c} \frac{S_x}{t^*}\,dz' + \frac{1}{2AGS}\oint_S \frac{h_Y}{t}\,ds \cdot \int_{z'}^{c} \frac{S_y}{t^*}\,dz'$$
$$- \frac{1}{4A^2G}\sum_i \left\{T_{Bio}\left[\oint_S \frac{h_i}{t}\,ds\right]^2 \int_{z'}^{c} G_{ri}\,dz'\right\} \quad . \quad . \quad . \quad . \quad . \quad (353a)$$

To obtain the deflections u_E and v_E equation (296) may be used in the form,

$$\sum_i \left\{\frac{h_i}{t}\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_i}{dr}\right) + \frac{G}{E}\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)\frac{g_i}{\rho t_s^*}\right\}$$
$$+ \frac{h_B}{t}\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_B}{dr}\right)$$
$$+ \frac{h_X}{t}\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_X}{dr}\right) + \frac{G}{E}\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_X}{ds}\right)\frac{g_X}{\rho t_s^*}$$
$$+ \frac{h_Y}{t}\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_Y}{dr}\right) + \frac{G}{E}\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_Y}{ds}\right)\frac{g_Y}{\rho t_s^*}$$
$$= -G\left[p_E\frac{d}{dr}\left(\rho^2\frac{d\theta}{dr}\right) + \rho\sin\psi\frac{d^2u_E}{dr^2} + \rho\cos\psi\frac{d^2v_E}{dr^2}\right] . \quad . \quad (354)$$

Multiplying equation (354) by h_x, integrating over all walls and using equations (280a), (282), (284a), (285b) and (297) one obtains,

$$-\frac{S}{2A}\sum_i\left\{(\eta_i-\eta_E)\oint_S\frac{h_i}{t}\,ds\,.\,\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_i}{dr}\right)\right\}$$
$$+\oint_S\frac{h_X}{t}\,ds\,.\,\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_B}{dr}\right)$$
$$+\oint\frac{h_X^2}{t}\,ds\,.\,\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_X}{dr}\right)-\frac{G}{E}\frac{S^2}{I_x}\frac{g_X}{\rho t_s^*}$$
$$+\oint\frac{h_Xh_Y}{t}\,ds\,.\,\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_Y}{dr}\right)=G\rho S\frac{d^2u_E}{dr^2}\quad.\quad.\quad.\quad.\quad.\quad(355)$$

Dividing by GS, integrating with respect to r and using equations (268), (275), (275a), (277) and (277a), equation (355) reduces to,

$$\rho^2\frac{d}{dr}\left(\frac{u_E}{\rho}\right)=-\frac{1}{EI_y}\int_{r_o}^{r}\frac{M_y}{\rho^2t_s^*}\,dr+\frac{1}{GS^2}\oint\frac{h_X^2}{t}\,ds\,.\,\frac{Q_x}{t^*}$$
$$+\frac{1}{GS^2}\oint\frac{h_Xh_Y}{t}\,ds\,.\,\frac{Q_y}{t^*}-\frac{1}{2AGS}\oint_S\frac{h_X}{t}\,ds\,.\,\frac{T_{BE}}{\rho t^*}$$
$$-\frac{1}{2AG}\sum_i\left\{(\eta_i-\eta_E)\oint_S\frac{h_i}{t}\,ds\,.\,\left(\frac{\rho}{t^*}\frac{dg_i}{dr}\right)\right\}+\text{const.}\quad(356)$$

To find the constant, equation (309) of 5.4.4 may be used. Thus, at a section with $\theta=0$,

$$\frac{\partial w}{\partial s}=-\frac{\rho}{Gt^*}\left[\frac{h_B}{t}\frac{dg_B}{dr}+\frac{h_X}{t}\frac{dg_X}{dr}+\frac{h_Y}{t}\frac{dg_Y}{dr}+\sum_i\frac{h_i}{t}\frac{dg_i}{dr}\right]$$
$$-p_E\rho^2\frac{d\theta}{dr}-\rho\sin\psi\,\frac{du_E}{dr}-\rho\cos\psi\,\frac{dv_E}{dr}\quad.\quad.\quad.\quad.\quad.\quad(309a)$$

Multiplying equation (309a) by h_X and integrating over all walls one obtains,

$$\rho S\frac{du_E}{dr}=\frac{1}{GS}\oint\frac{h_X^2}{t}\,ds\,.\,\frac{Q_x}{t^*}+\frac{1}{GS}\oint\frac{h_Xh_Y}{t}\,ds\,.\,\frac{Q_y}{t^*}$$
$$-\frac{1}{2AG}\oint_S\frac{h_X}{t}\,ds\,.\,\frac{T_{BE}}{\rho t^*}-\frac{S}{2AG}\sum_i\left\{(\eta_i-\eta_B)\oint_S\frac{h_i}{t}\,ds\,.\,\left(\frac{\rho}{t^*}\frac{dg_i}{dr}\right)\right\}+\oint h_X\frac{\partial w}{\partial s}\,ds\quad(357)$$

For a built-in tube, $\left[\frac{\partial w}{\partial s}\right]_{r=r_o}=0$, so that,

$$\left[\frac{du_E}{dr}\right]_{r=r_o}=\frac{1}{GS^2}\oint\frac{h_X^2}{t}\,ds\,.\,Q_{xo}+\frac{1}{GS^2}\oint\frac{h_Xh_Y}{t}\,ds\,.\,Q_{yo}-\frac{1}{2AGS}\oint_S\frac{h_X}{t}\,.\,T_{BEo}$$
$$-\frac{1}{2AG}\sum_i\left\{(\eta_i-\eta_E)\oint_S\frac{h_i}{t}\,ds\,.\left[\frac{dg_i}{dr}\right]_{r=r_o}\right\}\quad.\quad.\quad.\quad.\quad(357a)$$

Hence since $[u_E]_{r=r_o}=0$ the constant of integration in (356) vanishes.

Dividing equation (356) by ρ^2, integrating with respect to r, multiplying by ρ, noting that

$$\rho\int_{r_o}^{r}\frac{1}{\rho^2}\int_{r_o}^{r}\frac{M_y}{\rho^2t_s^*}\,dr=\int_{r_o}^{r}\int_{r_o}^{r}\frac{M_y}{\rho^3t_s^*}\,drdr\quad.\quad.\quad.\quad.\quad.\quad(358)$$

and using equations (315), the deflection u_E is given by,

$$u_E = \frac{1}{EI_y}\int_{r_0}^{r}\int_{r_0}^{r}\frac{M_y}{\rho^3 t_s^*}\,dr + \frac{1}{GS^2}\oint\frac{h_X^2}{t}\,ds\,.\,\rho\int_{r_0}^{r}\frac{Q_x}{\rho^2 t^*}\,dr + \frac{1}{GS^2}\oint\frac{h_X h_Y}{t}\,ds\,.\,\rho\int_{r_0}^{r}\frac{Q_y}{\rho^2 t^*}\,dr$$

$$- \frac{1}{2AGS}\oint_S\frac{h_X}{t}\,ds\,.\,\rho\int_{r_0}^{r}\frac{T_{BE}}{\rho^3 t^*}\,dr + \frac{1}{4A^2G}\sum_i\left\{T_{Bio}(\eta_i-\eta_E)\left[\oint_S\frac{h_i}{t}\,ds\right]^2\rho\int_{r_0}^{r}\frac{G_{ri}}{\rho}\,dr\right\} \quad (359)$$

For a cylindrical tube equation (359) reduces to,

$$u_E = -\frac{1}{EI_y}\int_c^{z'}\int_c^{z'}\frac{M_y}{t_s^*}\,dz' + \frac{1}{GS^2}\oint\frac{h_X^2}{t}\,ds\,.\int_c^{z'}\frac{S_x}{t^*}\,dz' + \frac{1}{GS^2}\oint\frac{h_X h_Y}{t}\,ds\,.\int_c^{z'}\frac{S_y}{t^*}\,dz'$$

$$- \frac{1}{2AGS}\oint_S\frac{h_X}{t}\,ds\,.\int_c^{z'}\frac{T_{BE}}{t^*}\,dz' + \frac{1}{4A^2G}\sum_i\left\{T_{Bio}(\eta_i-\eta_E)\left[\oint_S\frac{h_i}{t}\,ds\right]^2\int_c^{z'}G_{ri}\,dz'\right\} \quad (359a)$$

The first four terms on the R.H.S. of equations (359) represent the deflections due to the ordinary engineers' theory. Thus the first term may be written,

$$-\int_{r_0}^{r}\int_{r_0}^{r}\frac{M_y}{EI_y}\,dr$$

and is clearly the usual expression for deflection due to bending. The second and third terms are the engineers' theory shear deflections in the X-direction due to the shear stress resultants in the X and Y directions respectively. The fourth term is the deflection of the line OE due to the torque T_{BE} and vanishes if E is the shear centre E_s.

The last expression on the R.H.S. of equations (359) is the correction to the deflection arising from axial constraints. For a tube with an axis of symmetry in the X-direction the pure flexural modes in this direction will satisfy condition (335),

$$\oint_S\frac{h_i}{t}\,ds = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (335)$$

while the corresponding η_i's become infinite. For these modes the expression

$$T_{Bio}(\eta_i-\eta_E)\left[\oint_S\frac{h_i}{t}\,ds\right]^2$$

should be replaced by

$$Q_{xo}(a_{yi})^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (359b)$$

where,

$$a_{yi} = \eta_i\oint_S\frac{h_i}{t}\,ds \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (360)$$

To obtain the appropriate expressions for v_E the suffices x and y in the first four terms of equations (359) and (359a) should be interchanged, and in the last terms of the latter equations $-(\eta_i-\eta_E)$ should be replaced by $+(\xi_i-\xi_E)$. In the limiting form of equation (359b) x and y should be interchanged.

By working in terms of axes GX_ϕ, GY_ϕ inclined at an angle ϕ to the principal axes such that,

$$\tan 2\phi = \frac{2\oint\frac{h_X h_Y}{t}\,ds}{\oint\frac{h_Y^2}{t}\,ds - \oint\frac{h_X^2}{t}\,ds}$$

the term in u_E involving Q_{yo} and the term in v_E involving Q_{xo} will vanish. The new axes may be called *the principal axes in shear* since for a load in the direction of one of the axes there is no deflection due to engineers' theory shear in the other direction. However, there is no advantage in using these axes since the bending deflections due to engineers' theory are no longer orthogonal. In sections with an axis of symmetry the principal shear axes have the same directions as the principal (bending) axes.

The axial constraint corrections to the deformations due to the ordinary engineers' theory will be in general of a very small order compared with the corresponding corrections to the stresses.

5.4.15. *Formulæ for stresses and deflections referred to any system of axes XY.*

It is sometimes convenient to use an orthogonal system of axes X, Y different from the principal axes in bending. All the formulæ for the stresses and deflections previously derived can still be applied by making the following substitutions,

$$\left.\begin{aligned} \bar{Q}_x &= \frac{Q_x - \{ I_{xy}/I_x \} Q_y}{1 - \{ (I_{xy})^2/I_x I_y \}} \quad \text{for } Q_x \\ \bar{M}_y &= \frac{M_y - \{ I_{xy}/I_x \} M_x}{1 - \{ (I_{xy})^2/I_x I_y \}} \quad \text{for } M_y \end{aligned}\right\} \quad \text{and} \quad \left.\begin{aligned} \bar{Q}_y &= \frac{Q_y - \{ I_{xy}/I_y \} Q_x}{1 - \{ (I_{xy})^2/I_x I_y \}} \quad \text{for } Q_y \\ \bar{M}_x &= \frac{M_x - \{ I_{xy}/I_y \} M_y}{1 - \{ (I_{xy})^2/I_x I_y \}} \quad \text{for } M_x \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (361)$$

It should be noted that *all* the quantities in the above equations and the formulæ for the stresses and deflections are referred to the new system of axes X, Y. Thus, for example, to obtain the appropriate h_X-function equation (275) must be used with I_y and $\bar{x}$ referred to the *new* axes.

5.4.16. *The warping function W_i^* for arbitrary loading.*

The warping of a four-boom tube has been expressed, in 4.3.8, in terms of a single warping function W^*. In the general case it is no longer possible to express the warping in terms of a single function, but the warping is expressible in terms of a series of functions W_i^* corresponding to each eigenvalue λ_i. When dealing with boundary conditions involving warping, such as at the joint between two tubes, it is always possible to work in terms of equation (26a); this procedure has, for example, been used to establish equation (313). Since, however, $\frac{\partial w}{\partial s}$ is altered by the addition of rigid body movements of a cross-section, which do not affect the actual warping, it is desirable to introduce the warping functions W_i^* which exclude *ab initio* any rigid body movements *independently of the axis of reference OE*.

The functions W_i^* are defined by the equations,

$$W_i^* = \oint h_i \frac{\partial w}{\partial s} ds = - \oint \frac{dh_i}{ds} w ds \quad . \quad . \quad . \quad . \quad (362)$$

or for the n-boom single-cell tube,

$$W_i^* = - \sum_{m=1}^{n} w_m [(h_i)_{m,\, m+1} - (h_i)_{m-1,\, m}] \quad . \quad . \quad . \quad (362a)$$

Thus, if at the end of a tube the displacement w is specified, the warping functions W_i^* are determined by equations (362).

The warping function W^* of 4.3.8 is related to the unique function $W_1{}^*$ for the four-boom tube by the relation,

$$W^* = -2\rho^2 \left[\sum_1^4 \frac{s_{12}}{t_{12}} (\bar{A}_{34})^2 \right]^{1,2} W_1{}^* \quad . \quad . \quad . \quad . \quad (363)$$

The functions $W_i{}^*$ for the general loading case will now be investigated. Substituting from equation (26a) for $\frac{\partial w}{\partial s}$ in equation (362), and using equations (118), (284a), (284b), (57), and (54), one obtains,

$$W_i{}^* = -\frac{\rho}{Gt^*} \left[\frac{dg_B}{dr} - \frac{(\eta_i - \eta_E) S}{2A} \frac{dg_X}{dr} + \frac{(\xi_i - \xi_E) S}{2A} \frac{dg_Y}{dr} \right] \oint_S \frac{h_i}{t} ds - \frac{\rho}{Gt^*} \frac{dg_i}{dr} \quad . \quad (364)$$

or with equations (268), (277), (277a), (300), and (315),

$$W_i{}^* = \frac{1}{2AG} \oint_S \frac{h_i}{t} ds \, . \left\{ \frac{T_{Bi}}{\rho t^*} + T_{Bio} \rho G_{ri} \right\} \quad . \quad . \quad . \quad . \quad (365)$$

For the cylindrical tube put $\rho = 1$. In the exceptional case discussed in 5.4.12 equation (365) becomes,

$$W_i{}^* = -\frac{a_{xi}}{2AG} \left\{ \frac{Q_y}{t^*} + Q_{yo} \rho^2 G_{ri} \right\} . \quad . \quad . \quad . \quad . \quad (365a)$$

It now appears that the boundary condition (313) for warping at a discontinuity in T_{Bi}, t^*, and/or $t_s{}^*$ at $r = r_1$ may be concisely expressed in the form,

$$[W^*{}_{iR}]_{r=r_1} = [W^*{}_{iL}]_{r=r_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (366)$$

Thus the condition for equality of warping takes an analogous form to the condition of equality of direct stress flow (312).

5.5. STRESSES AND WARPING DUE TO AN ARBITRARY SELF-EQUILIBRATING END LOAD SYSTEM.

In this section the stresses and warping due to an arbitrary self-equilibrating end load system applied at either end of a tube will be investigated. It will be shown that the stresses and warping functions for such a load system may be expressed in terms of unit eigenload systems typified by σ_i, τ_i.

Thus,

$$\sigma = \sum_i e_i \sigma_i = \sum_i e_i \mu_i G_{\sigma i} L_{\sigma i} \quad . \quad . \quad . \quad . \quad . \quad (376)$$

$$\tau = \sum_i e_i \tau_i = \sum_i e_i \mu_i G \quad L_{\tau i} \quad . \quad . \quad . \quad . \quad . \quad (379)$$

and,

$$W_i{}^* = e_i \, \overline{W}_i{}^* = c_i \frac{\mu_i}{G} \rho G_{\tau i} \quad . \quad . \quad . \quad . \quad . \quad (381)$$

where the L-functions are given by equations (340) and the G-functions are proportional to the longitudinal variation of the stresses. The e_i-coefficients are analogous to "Fourier-coefficients" and are obtained from the given end stress distribution.

5.5.1. *The unit eigenloads or end load systems.*

The stresses due to an eigenload system in the mode i are,

$$\left.\begin{aligned} \sigma_i &= \left(\frac{1}{t_s} \frac{dh_i}{ds} \right) \frac{g_i}{\rho t_s{}^*} \\ \tau_i &= -\frac{h_i}{t} \frac{1}{t^*} \frac{dg_i}{dr} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (367)$$

where g_i is given by equation (89),

$$g_i = \overline{D}_{1i} \Psi_{1i}(\rho) + \overline{D}_{2i} \Psi_{2i}(\rho) \quad . \quad . \quad . \quad . \quad . \quad (89a)$$

The coefficients $\bar{D}$ depend on the magnitude of the stresses at the end of the tube (see 3.2.4).

It is useful for practical purposes to define "unit" end load systems of two types. In the first the root section ($\rho=1$) is loaded with the tip ($\rho=\rho_c$) free, while in the second the tip is loaded with the root free. The magnitudes of these unit eigenload systems are in both cases defined by,

and,
$$\left.\begin{aligned}[g_i]_L &= 1\\ [g_i]_F &= 0\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (368)$$

where the suffices L and F refer to the loaded and free end of the tube. Note that g_i has the dimensions of a load.

For the unit end load systems equations (367) may be written,

$$\left.\begin{aligned}\sigma_i &= \mu_i L_{\sigma i} G_{\sigma i}\\ \tau_i &= \mu_i L_{\tau i} G_{\tau i}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (369)$$

where the L-functions are given in equations (340) and the G-functions are as follows,

$$\left.\begin{aligned}G_{\sigma i} &= \frac{g_i}{\rho t_s^*}\\ G_{\tau i} &= -\frac{1}{\mu_i t^*}\frac{dg_i}{dr}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (370)$$

in which g_i is the unit function defined by equations (368).

From equations (35) and (95) the rate of twist and the warping are given by,

$$\frac{d\theta_i}{dr} = \frac{1}{2AG}\mu_i \oint_S \frac{h_i}{t}\,ds\,.\,\frac{G_{\tau i}}{\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad (371)$$

and,

$$w_i^* = -\frac{1}{E} L_{\sigma i}\rho G_{\tau i} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (372)$$

The warping function $\bar{W}_i^*$ for the unit eigenload system in the ith mode may be obtained by substituting equation (372) in the last of equations (362), thus

$$\bar{W}_i^* = \frac{\mu_i}{G}\rho G_{\tau i} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (372a)$$

Note that under the ith unit end load the tube twists about the axis OS_i. If $\oint_S \frac{h_i}{t}\,ds = 0$ while S_i is at infinity the tube is subject to bending without twist. In this case the deflections are given by,

$$\left.\begin{aligned}\frac{d}{dr}\left(\frac{u_i}{\rho}\right) &= \frac{1}{2AG} a_{yi}\mu_i \frac{G_{\tau i}}{\rho}\\ \frac{d}{dr}\left(\frac{v_i}{\rho}\right) &= -\frac{1}{2AG} a_{xi}\mu_i \frac{G_{\tau i}}{\rho}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (373)$$

If in addition S_i is at finitum the ith end load system produces pure warping, *i.e.* no bend or twist.

For the particular case of a conical tube with $t^* = t_s^* = \rho^\alpha$

$$g_i = \rho^{\alpha/2}[\bar{D}_{1i}\rho^{\nu_i} + \bar{D}_{2i}\rho^{-\nu_i}] \quad . \quad . \quad . \quad . \quad . \quad (86c)$$

where

$$\nu_i = \frac{1}{2}[\alpha^2 + 4\mu_i{}^2 r_o{}^2]^{1/2} \quad . \quad . \quad . \quad . \quad . \quad (86a)$$

From equation (86c) and conditions (368) one obtains for the G-functions of the unit eigenload systems,

for load at root,

$$\left.\begin{aligned} G_{\sigma i} &= \rho^{-(\alpha+2)/2}\,\frac{(\rho/\rho_e)^{\nu_i}-(\rho/\rho_e)^{-\nu_i}}{\rho_e^{-\nu_i}-\rho_e^{\nu_i}} \\ G_{\tau i} &= -\frac{1}{\mu_i r_o}\,\rho^{-(\alpha+2)/2}\,\frac{\left(\frac{\alpha}{2}+\nu_i\right)(\rho/\rho_e)^{\nu_i}-\left(\frac{\alpha}{2}-\nu_i\right)(\rho/\rho_e)^{-\nu_i}}{\rho_e^{-\nu_i}-\rho_e^{\nu_i}} \end{aligned}\right\} \quad . \quad . \quad (374)$$

and for load at tip,

$$\left.\begin{aligned} G_{\sigma i} &= \rho^{-(\alpha+2)/2}\,\frac{\rho^{-\nu_i}-\rho^{\nu_i}}{\rho_e^{-\nu_i}-\rho_e^{\nu_i}} \\ G_{\tau i} &= -\frac{1}{\mu_i r_o}\,\rho^{-(\alpha+2)/2}\,\frac{\left(\frac{\alpha}{2}-\nu_i\right)\rho^{-\nu_i}-\left(\frac{\alpha}{2}+\nu_i\right)\rho^{\nu_i}}{\rho_e^{-\nu_i}-\rho_e^{\nu_i}} \end{aligned}\right\} \quad . \quad . \quad (375)$$

In the limiting case of a uniform cylindrical tube the G-functions become,

for load at root,

$$\left.\begin{aligned} G_{\sigma i} &= \frac{\sinh \mu_i z'}{\sinh \mu_i l} \\ G_{\tau i} &= -\frac{\cosh \mu_i z'}{\sinh \mu_i l} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (374a)$$

and for a load at tip,

$$\left.\begin{aligned} G_{\sigma i} &= \frac{\sinh \mu_i (l-z')}{\sinh \mu_i l} \\ G_{\tau i} &= \frac{\cosh \mu_i (l-z')}{\sinh \mu_i l} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (375a)$$

Typical plots of the functions G_σ and G_τ for a 2:1 taper conical tube with $\alpha=0$, and for a uniform cylindrical tube are given in Figs. 20, 21 and 22. For the notation, see p. 239 of 4.1.4.

5.5.2. *Development of an arbitrary self-equilibrating end load system in terms of unit end load systems.*

An arbitrary self-equilibrating end load system may be expressed as a linear combination of unit eigenload systems. Thus, the direct stress σ may be written,

$$\sigma = \sum_i e_i \sigma_i = \sum_i e_i \mu_i L_{\sigma i} G_{\sigma i} \quad . \quad . \quad . \quad . \quad . \quad (376)$$

where e_i are the "Fourier-coefficients" of the expansion. Rewriting expression (376) for the loaded end of the tube one obtains from equations (368) and (370),

$$\sigma_L = \frac{1}{[\rho t_s^*]_L} \sum_i e_i \mu_i L_{\sigma i} \quad . \quad . \quad . \quad . \quad . \quad (377)$$

where σ_L defines the given self-equilibrating end load system.

Multiplying equation (377) by $\frac{dh_i}{ds}$ integrating over all walls and using relations (340), (77) and (78),

$$e_i = \frac{[\rho t_s^*]_L}{\lambda_i^2} \oint \sigma_L \frac{dh_i}{ds}\, ds \quad . \quad . \quad . \quad . \quad . \quad (378)$$

or

$$e_i = \frac{\rho_L}{\lambda_i^2} \oint f_L \left(\frac{1}{t_s} \frac{dh_i}{ds}\right) ds \quad . \quad . \quad . \quad . \quad . \quad (378a)$$

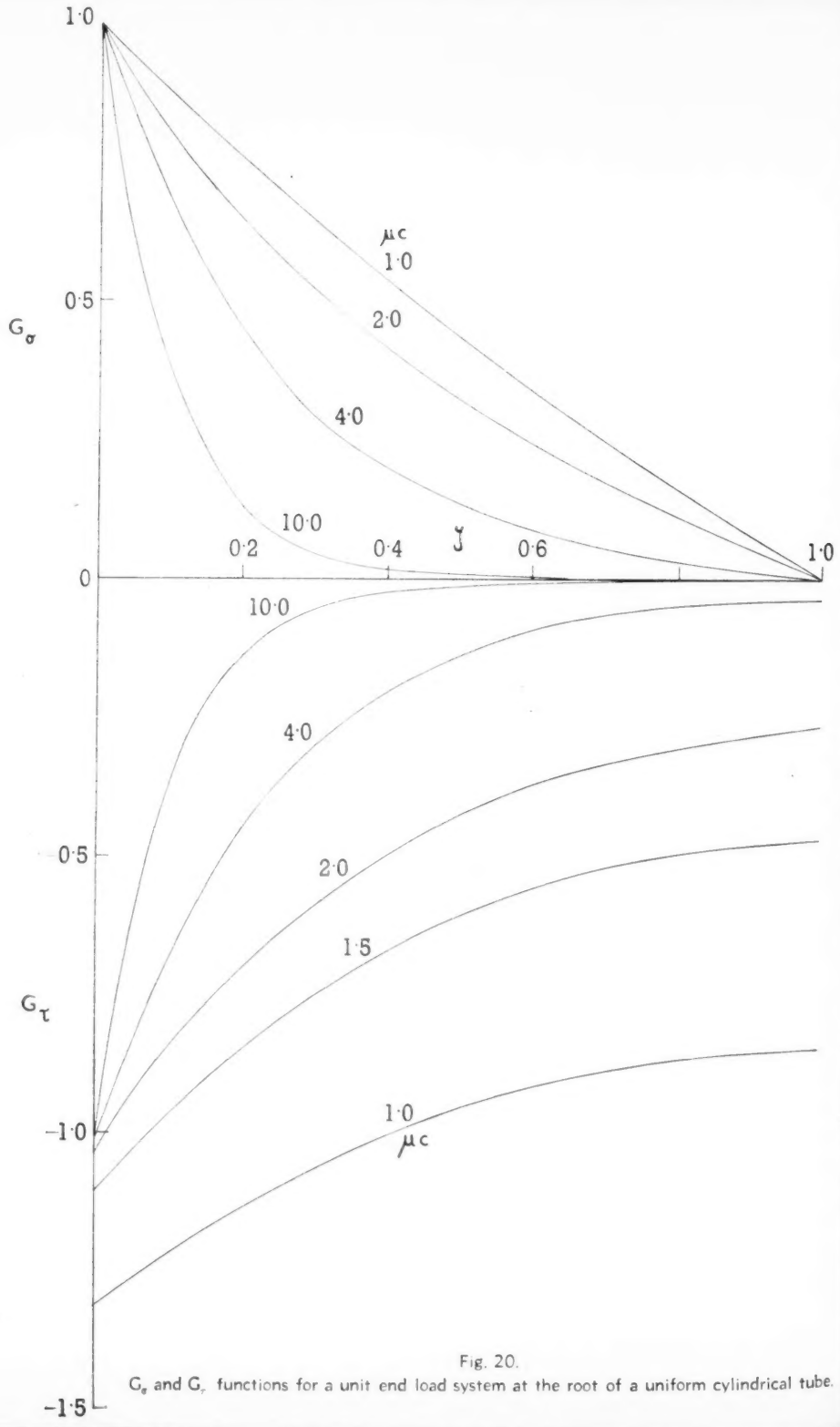


Fig. 20.
G_σ and G_τ functions for a unit end load system at the root of a uniform cylindrical tube.

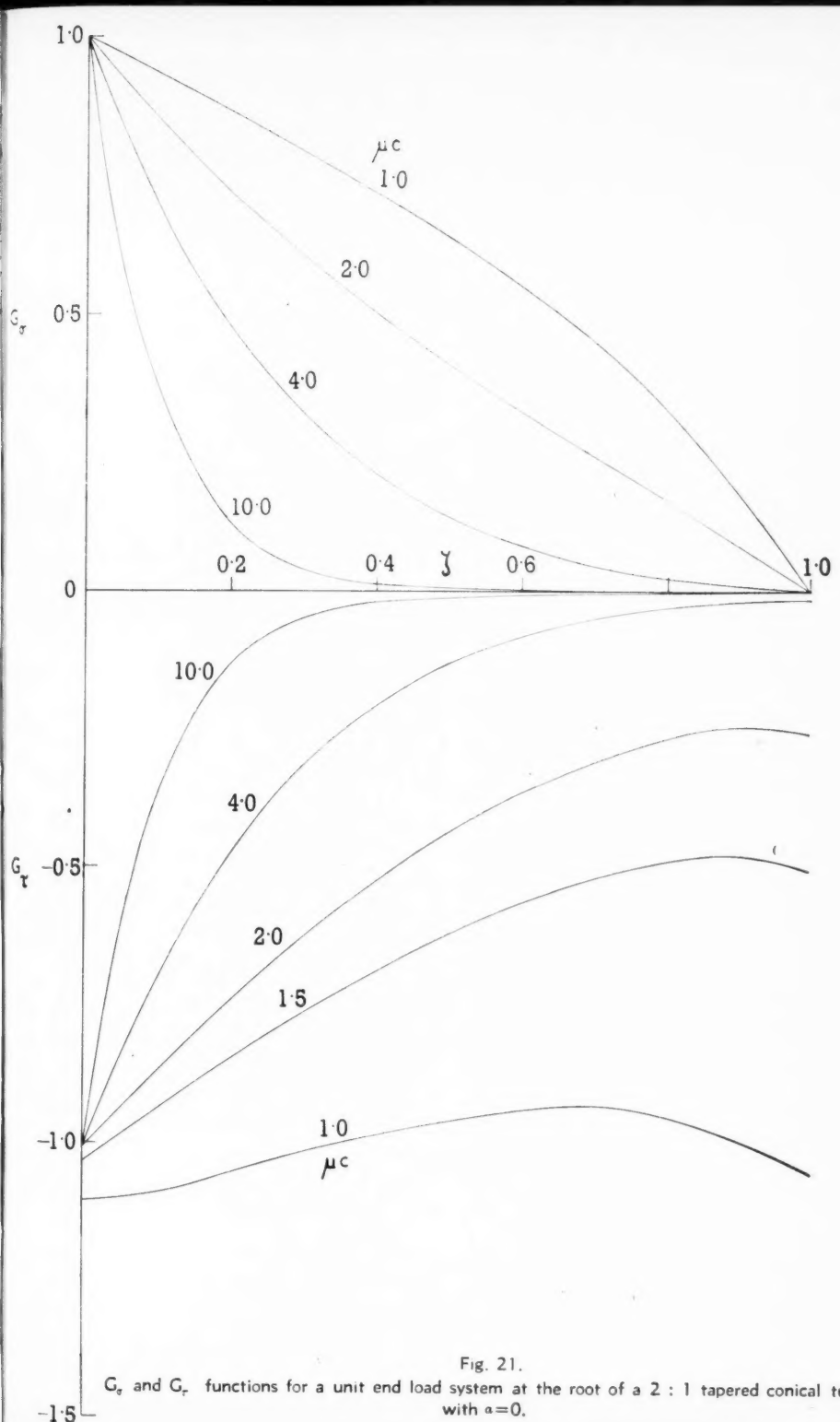


Fig. 21.

G_σ and G_τ functions for a unit end load system at the root of a 2 : 1 tapered conical tube with $\alpha=0$.

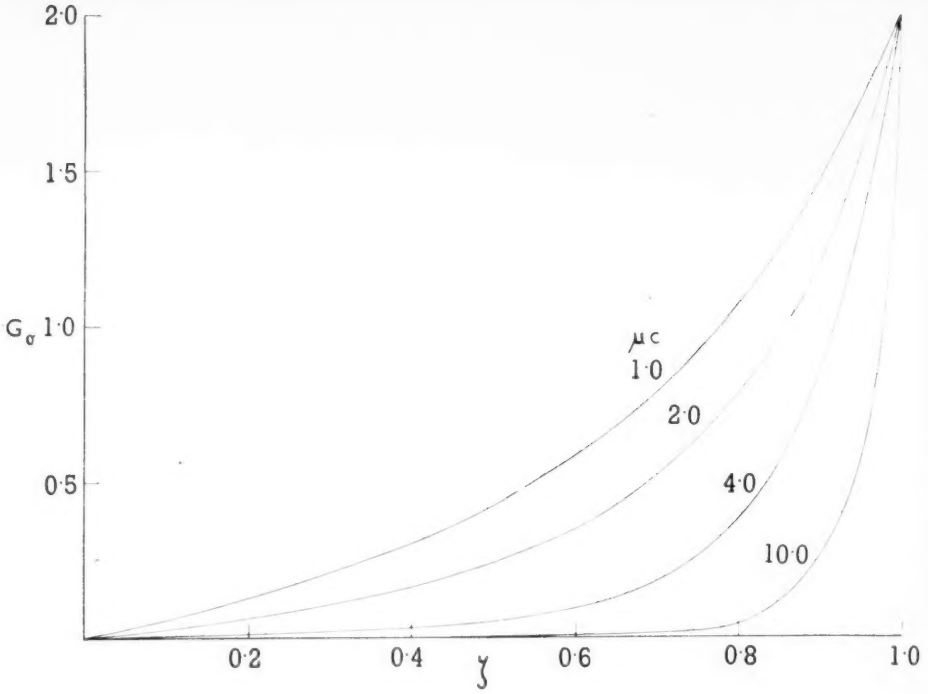
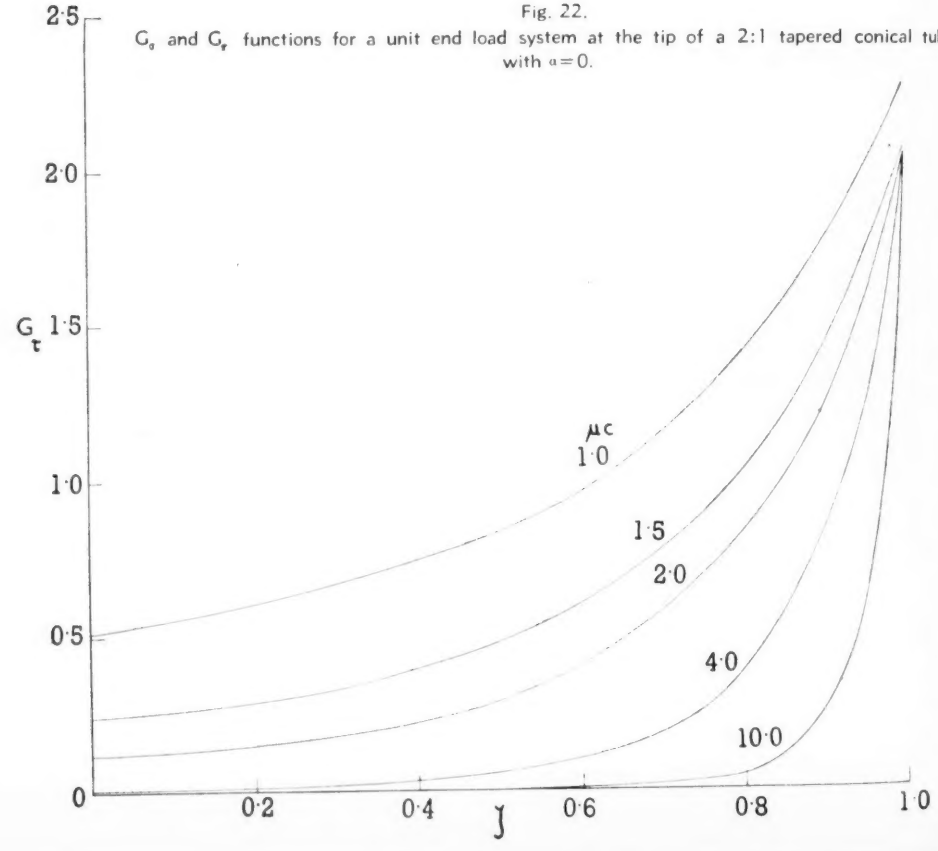


Fig. 22.
G_σ and G_τ functions for a unit end load system at the tip of a 2:1 tapered conical tube with $\alpha = 0$.



where f_L is the direct stress flow at the loaded section. An alternative form of equation (378a) is,

$$e_i = \frac{\rho_L}{\mu_i}\frac{G}{E}\oint f_L L_{\sigma i} ds \quad . \quad . \quad . \quad . \quad . \quad (378b)$$

For an n-boom tube with walls effective only in shear equation (378b) becomes,

$$e_i = \frac{1}{\mu_i}\frac{G}{E}\sum_{m=1}^{n} P_{mL} L_{\sigma i} \quad . \quad . \quad . \quad . \quad . \quad . \quad (378c)$$

where P_{mL} is the given load on the mth boom at the loaded section.

The shear stress distribution associated with equation (376) is,

$$\tau = \sum_i e_i \tau_i = \sum_i e_i \mu_i L_{\tau i} G_{\tau i} \quad . \quad . \quad . \quad . \quad . \quad (379)$$

From equations (371) and (373) the rate of twist and the warping functions are,

$$\frac{d\theta}{dr} = \sum_i e_i \frac{d\theta_i}{dr} = \frac{1}{2AG}\sum_i e_i \mu_i \oint_S \frac{h_i}{t} ds \,.\, \frac{G_{\tau i}}{\rho} \quad . \quad . \quad . \quad . \quad (380)$$

and,

$$W_i^* = e_i \overline{W}_i^* = e_i \frac{\mu_i}{G}\rho G_{\tau i} \quad . \quad . \quad . \quad . \quad . \quad (381)$$

The deflections u_E and v_E of the point E' which lies on the line OE through the apex, will be given by,

$$\left.\begin{aligned} \frac{d}{dr}\left(\frac{u_E}{\rho}\right) &= \sum_i e_i (\eta_i - \eta_E)\frac{d\theta_i}{dr} = \frac{1}{2AG}\sum_i e_i \mu_i (\eta_i - \eta_E)\oint_S \frac{h_i}{t} ds \,.\, \frac{G_{\tau i}}{\rho} \\ \frac{d}{dr}\left(\frac{v_E}{\rho}\right) &= -\sum_1 e_i (\xi_i - \xi_E)\frac{d\theta_i}{dr} = -\frac{1}{2AG}\sum_i e_i \mu_i (\xi_i - \xi_E)\oint_S \frac{h_i}{t} ds \,.\, \frac{G_{\tau i}}{\rho} \end{aligned}\right\} \quad . \quad (382)$$

Equations (382) presume that in each eigenload i the tube twists about the line OS_i. The proper procedure when in a mode i, $\oint_S \frac{h_i}{t} ds = 0$, follows immediately from equations (373).

5.6. The four-boom tube under arbitrary loading.

The results developed in 5.4 and 5.5 for the general case are directly applicable to the four-boom tube by simply omitting the summation sign and suffix i in the various formulæ. Since, however, the four-boom tube is of special importance for practical calculations some of the results will be recapitulated. It will be shown that in general a line exists through which transverse loads give stresses and deflections strictly in accordance with the ordinary engineers' theory. This line is in general different from the flexural axis through the shear centre which is often erroneously used to separate bending from torsion.

5.6.1. *Stresses and deflections of four-boom tube under arbitrary loading.*

Expressions (314) for the stresses reduce in the present case to,

$$\left.\begin{aligned} \sigma &= \sigma_E + \frac{T_{B10}}{2A} G_\sigma H_\sigma \\ \tau &= \tau_B + \tau_E + \frac{T_{B10}}{2A} G_\tau H_\tau \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (383)$$

The H_σ and H_τ functions are given in equations (237), (238) and (239) of 4.3.5.

It follows from equations (383) that the stresses in a four-boom tube under arbitrary transverse loading are composed of three parts namely the Bredt-Batho shear distribution, the

ordinary engineers' theory stresses, and a *single* correction term arising from a torque expression T_{B1} about the line OS_1 where S_1 has the co-ordinates ξ, η at the root. The stresses due to T_{B1} have been investigated in 4.3 and need, in general, no further consideration. In practice an economy in calculation is effected by taking E coincident with S_1, so that $T_{BE}=T_{B1}$.

The twist and deflections are immediately given by equations (353), (359) and (360), the axial constraint corrections reducing in each case to a single term involving T_{B1_0}. The single warping function W_1^* follows at once from equation (365),

$$W_1^*=\frac{1}{2AG}\sum_1^4 \delta_{12}h_{12}\cdot\left\{\frac{T_{B1}}{\rho t^*}+T_{B1_0}\rho G_\tau\right\}. \quad . \quad . \quad . \quad . \quad (384)$$

When the transverse loads pass through the axis OS_1, $T_{B1}=0$, and hence the ordinary engineers' theory holds throughout the tube. If in addition the loads pass through the shear centre the tube will deflect without twist.

The reader will note that if T_{B1} is given by equation (303),

$$T_{B1}=T_{B1_0}\rho t^* \quad . \quad . \quad . \quad . \quad . \quad . \quad (303a)$$

and the ends of the tube are free to warp, then $G_\tau=0$ and the warping W_1^* is constant, which agrees with the statement made in 5.4.4.

If S_1 is at finitum while

$$\sum_1^4 \delta_{12}h_{12}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (385)$$

then $H_\sigma=H_\tau=0$ and, from equations (383), the engineers' theory holds throughout the tube for any loading, which follows also from the fact that the warping is zero by equation (384).

The case when condition (385) is satisfied but in which S_1 is at infinity needs special consideration as shown in 5.4.12. If the loading is purely torque the warping is still zero by equation (385) and the stresses are in accordance with Bredt-Batho. In the presence of shear loads T_{B1} becomes infinite and the expressions for the stresses reduce to,

$$\left.\begin{aligned}\sigma&=\sigma_E+\frac{1}{2A}(Q_{x_0}a_y-Q_{y_0}a_x)G_\sigma L_\sigma\\ \tau&=\tau_B+\tau_E+\frac{1}{2A}(Q_{x_0}a_y-Q_{y_0}a_x)G_\tau L_\tau\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (386)$$

where L_σ and L_τ are given by equations (340) and from equation (336) etc.,

$$\left.\begin{aligned}a_x&=\xi\sum_1^4 \delta_{12}h_{12}\\ a_y&=\eta\sum_1^4 \delta_{12}h_{12}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (387)$$

The warping function becomes,

$$W_1^*=\frac{1}{2AG}\left\{\frac{(Q_xa_y-Q_ya_x)}{\rho t^*}+(Q_{x_0}a_y-Q_{y_0}a_x)\rho G_\tau\right\} \quad . \quad . \quad . \quad (384a)$$

This case is the special case, implied in the third paragraph on p. 204 of 1.2, when the axial constraint stresses cannot be determined as a correction due to the torque expression T_B by the methods of part 4. In such cases, in general it will be impossible for a built-in tube to sustain transverse loads with stresses in accordance with ordinary engineers' theory, except when all the loads are in the direction inclined at an angle

$$\tan^{-1}(a_x/a_y) \quad . \quad . \quad . \quad . \quad . \quad . \quad (388)$$

to the $\overline{Y}$ axis, for in this case

$$Q_x a_y - Q_y a_x = 0.$$

In sections symmetrical about the axis $G\overline{Y}$, $a_x = 0$, and the direction given by relation (388) coincides with the axis of symmetry.

The present results may be compared with those of 5.4.12.

5.6.2. *Formulæ for ξ and η.*

To conclude the present section explicit formulæ for ξ and η will be given. It is necessary only to give ξ and η for the equivalent single-cell tube of 4.3.2 since S_1 for a multi-cell tube is the same as for its equivalent single-cell tube.

The co-ordinates ξ and η for the four-boom tube may be obtained by the general method developed in 5.4.13. However, in the present case more elegant expressions for ξ and η may be derived from the fact that

$$-\lambda^2 \oint \frac{h}{t}\, ds/A, \quad -\lambda^2 \eta \oint \frac{h}{t}\, ds/2A, \text{ and } \lambda^2 \xi \oint \frac{h}{t}\, ds/2A$$

are proportional to the co-factors of the last three elements in any one row of the determinant of equation (186) (p. 255 of 4.3.1). By this procedure ξ and η referred to axes through the intersection of the diagonals are given by,

$$\left.\begin{aligned} \xi &= -\frac{\sum_1^4 \pm \left\{\frac{A_{234}}{B_1}[\delta_{12}\overline{A}_{34}\,|23,\,34|_x - \delta_{23}\overline{A}_{41}\,|34,\,42|_x + \delta_{31}\overline{A}_{12}\,|42,\,23|_x - \delta_{41}\overline{A}_{23}\,|23,\,34|_x]\right\}}{[\delta_{12}\overline{A}_{34} - \delta_{23}\overline{A}_{41} + \delta_{31}\overline{A}_{12} - \delta_{41}\overline{A}_{23}]\sum_1^4 \frac{(A_{234})^2}{B_1}} \\ \text{and} \\ \eta &= -\frac{\sum_1^4 \pm \left\{\frac{A_{234}}{B_1}[\delta_{12}\overline{A}_{34}\,|23,\,34|_y - \delta_{23}\overline{A}_{41}\,|34,\,42|_y + \delta_{31}\overline{A}_{12}\,|42,\,23|_y - \delta_{41}\overline{A}_{23}\,|23,\,34|_y]\right\}}{[\delta_{12}\overline{A}_{34} - \delta_{23}\overline{A}_{41} + \delta_{31}\overline{A}_{12} - \delta_{41}\overline{A}_{23}]\sum_1^4 \frac{(A_{234})^2}{B_1}} \end{aligned}\right\} \quad (389)$$

where,

$$|pq,\, rs|_x = \begin{vmatrix} A_{pq} & x_{pq} \\ A_{rs} & x_{rs} \end{vmatrix} \quad \text{and} \quad |pq,\, rs|_y = \begin{vmatrix} A_{pq} & y_{pq} \\ A_{rs} & y_{rs} \end{vmatrix} \quad . \quad . \quad (390)$$

A_{pq} always denotes the swept area between the points p and q as the radius vector moves *clockwise* to an observer at O; thus $A_{12} = A_{41} + A_{12}$, but $A_{24} = A_{23} + A_{34}$. The terms in the summation appearing in the numerators of equations (389) are found by cyclic changes and taken alternatively positive and negative, the first term in $\frac{1}{B_1}$ being positive. Thus the term for ξ in $\frac{1}{B_4}$ is obtained by adding 3 to each suffix and becomes,

$$-\frac{A_{123}}{B_4}[\delta_{41}\overline{A}_{23}\,|12,\,23|_x - \delta_{12}\overline{A}_{34}\,|23,\,31|_x + \delta_{23}\overline{A}_{41}\,|31,\,12|_x - \delta_{31}\overline{A}_{12}\,|12,\,23|_x]$$

In the exceptional case considered in 5.4.12 when

$$\oint_S \frac{h}{t}\, ds = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (335)$$

one has in place of equations (389)

$$\left.\begin{aligned} a_x &= \xi \oint \frac{h}{t}\,ds \\ &= -\frac{\sum_1^4 \pm \left\{\frac{A_{234}}{B_1}\left[\delta_{12}\bar{A}_{34}\,|23,\,34|_x - \delta_{23}\bar{A}_{41}\,|34,\,42|_x + \delta_{34}\bar{A}_{12}\,|42,\,23|_x - \delta_{41}\bar{A}_{23}\,|23,\,34|_x\right]\right\}}{\left[\sum_1^4 \delta_{12}(\bar{A}_{34})^2\right]^{1/2}\sum_1^4 \frac{(A_{234})^2}{B_1}} \\ a_y &= \eta \oint \frac{h}{t}\,ds \\ &= -\frac{\sum_1^4 \pm \left\{\frac{A_{234}}{B_1}\left[\delta_{12}\bar{A}_{34}\,|23,\,34|_y - \delta_{23}\bar{A}_{41}\,|34,\,42|_y + \delta_{34}\bar{A}_{12}\,|42,\,23|_y - \delta_{41}\bar{A}_{23}\,|23,34|_y\right]\right\}}{\left[\sum_1^4 \delta_{12}(\bar{A}_{34})^2\right]^{1/2}\sum_1^4 \frac{(A_{234})^2}{B_1}} \end{aligned}\right\} \quad (391)$$

Note that if $a_x = a_y = 0$ (*i.e.* S_1 at finitum) the engineers' theory holds always.

For the particular case of a singly symmetrical four-boom trapezoidal tube with walls effective only in shear (see Fig. 29), formulæ (389) reduce to,

$$\left.\begin{aligned} \xi &= 0 \\ \frac{\eta}{2a} &= \frac{B_2 s_{23} L_{23} - B_1 s_{41} L_{41}}{(s_{23}+s_{41})\left[B_1 (s_{41})^2 + B_2 (s_{23})^2\right]\left[2\delta_{12} s_{23} s_{41} - \delta_{23}(s_{41})^2 - \delta_{41}(s_{23})^2\right]} \end{aligned}\right\} \quad . \quad (392)$$

where

and

$$\left.\begin{aligned} L_{23} &= 2\delta_{12}(s_{23})^3 s_{41} + \delta_{23}(s_{41})^3(2s_{23}+s_{41}) - \delta_{41}(s_{23})^4 \\ L_{41} &= 2\delta_{12} s_{23}(s_{41})^3 + \delta_{41}(s_{23})^3(2s_{41}+s_{23}) - \delta_{23}(s_{41})^4 \end{aligned}\right\} \quad . \quad . \quad (392a)$$

In the exceptional case when $\oint \frac{h}{t}\,ds = 0$

$$\frac{a_y}{2a} = \frac{\eta}{2a}\oint \frac{h}{t}\,ds = \frac{B_2 s_{23} L_{23} - B_1 s_{41} L_{41}}{(s_{23}+s_{41})\left[B_1 (s_{41})^2 + B_2 (s_{23})^2\right]\left[2\delta_{12}(s_{23})^2 (s_{41})^2 + \delta_{23}(s_{41})^4 + \delta_{41}(s_{23})^4\right]^{\frac{1}{2}}} \quad (393)$$

It is instructive to compare the position of S_1, the shear centre for zero warping given by equation (392), with E_s, the shear centre for zero twist as defined by the ordinary engineers' theory.

The co-ordinates of E_s are given by,

$$\left.\begin{aligned} \xi_{ES} &= 0 \\ \frac{\eta_{ES}}{2a} &= \frac{\left[B_2 (s_{23})^3 - B_1 (s_{41})^3\right]\sum_1^4 \delta_{12} - (s_{23}+s_{41})^2\left[B_2\delta_{23} s_{23} - B_1\delta_{41} s_{41}\right]}{(s_{23}+s_{41})\left[B_1 (s_{41})^2 + B_2 (s_{23})^2\right]\sum_1^4 \delta_{12}} \end{aligned}\right\} \quad . \quad . \quad (394)$$

Note that in equations (392) and (394) η and η_{ES} are measured from the intersection of the diagonals. The difference between the two centres may be written in the form,

$$\frac{\eta - \eta_{ES}}{2a} = -\frac{(s_{23}+s_{41})}{B_1 (s_{41})^2 + B_2 (s_{23})^2}\left\{\frac{(B_1\delta_{41} s_{23} - B_2\delta_{23} s_{41})\, s_{23} s_{41}}{2\delta_{12} s_{23} s_{41} - \delta_{23}(s_{41})^2 - \delta_{41}(s_{23})^2} + \frac{B_1\delta_{41} s_{41} - B_2\delta_{23} s_{23}}{\sum_1^4 \delta_{12}}\right\} \quad (395)$$

If $2\delta_{12} s_{23} s_{41} - \delta_{23}(s_{41})^2 - \delta_{41}(s_{23})^2$, which is $\propto \oint \frac{h}{t}\,ds$, is zero then in general S_1 is at infinity and formula (393) has to be used. If, however, in addition,

$$B_1\delta_{41} s_{23} - B_2\delta_{23} s_{41} = 0$$

or

$$\frac{B_1}{t_{41}} = \frac{B_2}{t_{23}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (396)$$

then the position of S_1 is indeterminate, $a_y = 0$, and the engineers' theory is strictly true for any transverse loading of the four-boom tube with built-in root.

That the use of the shear centre E_s, as an axis of reference for obtaining the torque and hence the axial constraint stresses, might be attended with *certain* inaccuracies is shown clearly in Figs. 23 and 24. A perusal of these figures should convince even the most conservative reader that the flexural axis should be *terra interdita* when finding constraint stresses.

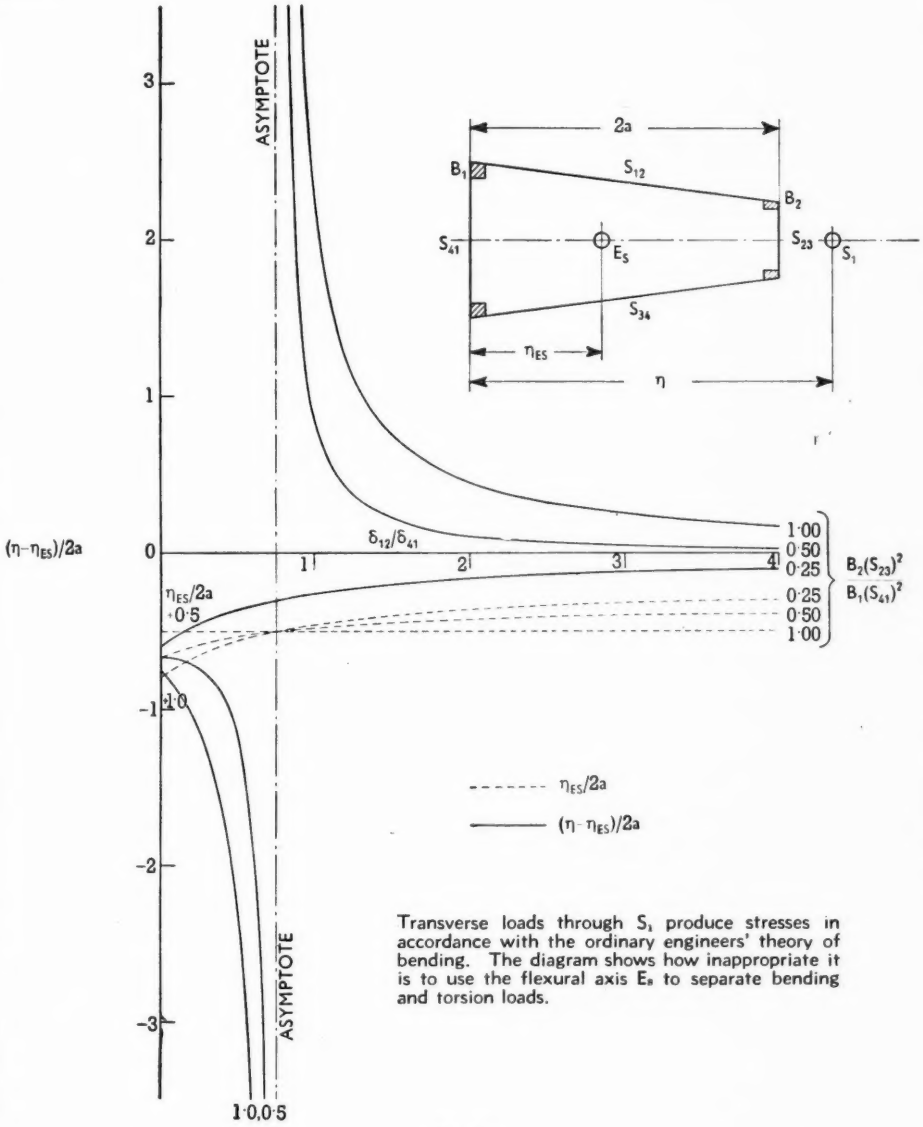
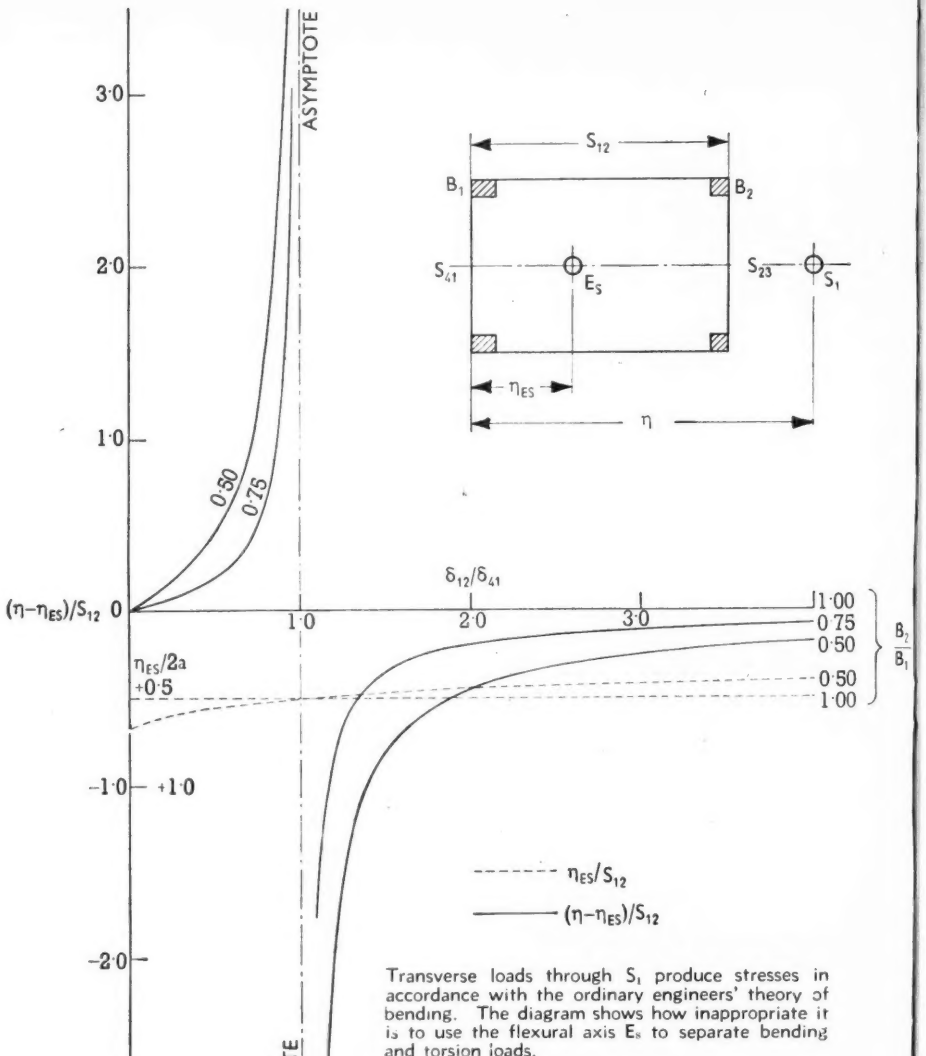


Fig. 23.

Comparison of shear centre E_s with centre for zero warping S_1 in singly symmetrical trapezoidal tube with $s_{41}=2s_{23}$ and $t_{41}=t_{23}$.



Transverse loads through S_1 produce stresses in accordance with the ordinary engineers' theory of bending. The diagram shows how inappropriate it is to use the flexural axis E_S to separate bending and torsion loads.

Fig. 24.

Comparison of shear centre E_S with centre for zero warping S_1 in singly symmetrical rectangular tube with $t_{41}=t_{23}$.

5.7. THE *n*-BOOM TUBE IN GENERAL.

It was stated in the summary and in 1.2 of the introduction that the six-boom tube would be analysed in part 5 of the paper. Since making this statement the writers have realised that the method they developed for the six-boom tube can be generalised for the *n*-boom tube. In fact, the proof of the secular equation and the *h*-functions in the *n*-boom tube is no more complex than that required for the special case $n=6$.

No theoretical limitation need be placed on the number of shear-carrying walls provided that the tube is of the type shown in Fig. 16, p. 263, *i.e.*, all walls connect adjacent booms. All such cases may be made to depend on the solution for the single-cell *n*-boom tube, of the type shown in Fig. 25, by an immediate generalisation of the method given in 4.3.2 for the four-boom tube. It will be shown that the analysis is much simplified when there are one or two axes of symmetry.

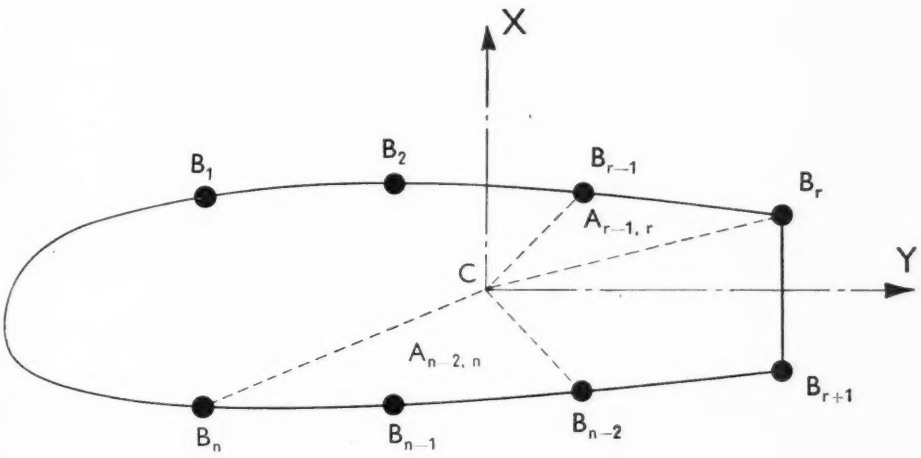


Fig. 25.
The n-boom single-cell tube of arbitrary cross-section.

5.7.1. *How to form the secular equation for the n-boom single-cell tube.*

In this section the rules for the formation of the secular equation will be given, the proof being postponed to the next paragraph. The formal expression for the secular equation may be found rather formidable, but with the aid of some simple geometrical rules it is easy to form and memorise this equation.

The secular equation Δ_n for the *n*-boomtube may be written in the form,

$$\Delta_n=(-1)^n\sum_{m=0}^{n-3}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+3}}\left\{\frac{1}{B_rB_s\ldots\ldots B_t}\sum_{\binom{m+3}{3}}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\,|ab,\,cd,\,ef|^2]\right\}=0 \quad (397)$$

where $r, s, \ldots\ldots t \neq i, j, \ldots\ldots q, a, b, \ldots\ldots f$ are boom numbers, and $\binom{n}{m+3}$ and $\binom{m+3}{3}$ are binomial coefficients denoting the number of terms under each summation. The abbreviation δ_{ij} denotes the sum

$$\delta_{ij}=\delta_{i,\,i+1}+\delta_{i+1,\,i+2}+\ldots\ldots+\delta_{j-1,\,j} \quad . \quad . \quad . \quad . \quad (398)$$

The expression $|ab, cd, ef|$ represents the determinant,

$$|ab, cd, ef| = \begin{vmatrix} A_{ab} & x_{ab} & y_{ab} \\ A_{cd} & x_{cd} & y_{cd} \\ A_{ef} & x_{ef} & y_{ef} \end{vmatrix} \quad . \quad . \quad . \quad . \quad (399)$$

Note that in forming δ_{ij}, A_{ab} and so on, one proceeds clockwise from i to j, a to b, and so on, and that if $i+m>n$ it becomes $(i+m-n)$.

The terms,

$$\frac{1}{B_r B_s \ldots\ldots B_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (400)$$

are formed from all possible products of $(n-m-3)$ different B's selected from the n-booms; thus there are in all $\binom{n}{m+3}$ such terms for the mth power of λ^2. It should be noted that the coefficient of the highest power in λ^2 does not involve any B.

To form the summation,

$$\sum_{\binom{m+3}{s}} [\delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq} |ab, cd, ef|^2] \quad . \quad . \quad . \quad . \quad (401)$$

under the term

$$\frac{1}{B_r B_s \ldots\ldots B_t}$$

one considers the tube with $(m+3)$ booms obtained by omitting the $(n-m-3)$ booms, B_r, B_s, ..., B_t from the original n-boom tube. Thus in the new $(m+3)$ boom tube only the booms at

$$i, j, \ldots\ldots q, a, b, \ldots\ldots f$$

will occur. In this tube one forms all possible products

$$\delta_{ij}\, \delta_{kl} \ldots\ldots \delta_{pq}$$

of m δ's selected from the $(m+3)$ walls, and multiplies each product by the square of the determinant

$$|ab, cd, ef|$$

corresponding to the three walls not occurring in the δ product. Thus to each term (400) the corresponding sum (401) will contain $\binom{m+3}{3}$ terms.

The total of terms in $(\lambda^2)^m$ will be,

$$\binom{n}{m+3}\binom{m+3}{3} = \frac{n!}{6m!\,(n-m-3)!} \quad . \quad . \quad . \quad . \quad (402)$$

and in the whole secular equation,

$$\frac{1}{3} n\,(n-1)\,(n-2)\,2^{n-4} \quad . \quad . \quad . \quad . \quad . \quad (403)$$

The constant term, *i.e.* the term for $m=0$ in (397),

$$(-1)^n \sum_{\binom{n}{3}} \left\{ \frac{1}{B_r B_s \ldots\ldots B_t} |ab, cd, ef|^2 \right\}$$

may be written in the somewhat simpler form,

$$(-1)^n\, 4A^2 \sum_{\binom{n}{3}} \frac{(A_{abc})^2}{B_r B_s \ldots\ldots B_t} \quad . \quad . \quad . \quad . \quad . \quad (404)$$

where $r \mp s \mp \ldots\ldots \mp t \mp a \mp b \mp c$.

The term involving the highest power of λ^2 may be written,

$$-(\lambda^2)^{n-3} \sum_{(3)}^{(n)} [\delta_{i,\,i+1}\,\delta_{k,\,k+1}\ldots\ldots\,\delta_{p,\,p+1}\,|a{\cdot}a+1,\ c{\cdot}c+1,\ e{\cdot}e+1|^2] \quad . \quad (405)$$

where $i \neq k \neq p \neq a \neq c \neq e$.

Note also that the term in λ^2 may be written,

$$(-1)^{n-1}\lambda^2 \sum_{(4)}^{(n)} \left\{ \frac{1}{B_r B_s \ldots\ldots B_t} \sum_{4} [\delta_{da}\,|ab,\ bc,\ cd|^2] \right\} \quad . \quad . \quad . \quad (406)$$

where $r \neq s \neq \ldots\ldots \neq t \neq a \neq b \neq c \neq d$.

As a check it is useful to know the maximum number of walls of the original tube that may occur in the expression δ_{ij} of equation (397). By the law of formation of expression (401) it follows that the maximum number of walls forming δ_{ij} is

$$(n-m-2)$$

the maximum of which occurs for $m=1$ and becomes $(n-3)$.

The determinants $|\,ab,\ cd,\ ef\,|$ may be obtained from the $\frac{1}{6}n\,(n-1)\,(n-2)$ independent determinants of the form $|\,a{\cdot}a+1,\ c{\cdot}c+1,\ e{\cdot}e+1\,|$. For example one would write,

$$|a{\cdot}a+s,\ c{\cdot}c+1,\ e{\cdot}e+1| = \sum_{r=0}^{s-1} |a+r{\cdot}a+r+1,\ c{\cdot}c+1,\ e{\cdot}e+1| \quad . \quad . \quad (407)$$

An illustration to the formation of the secular equation for the particular case of a six-boom tube is given in Fig. 26.

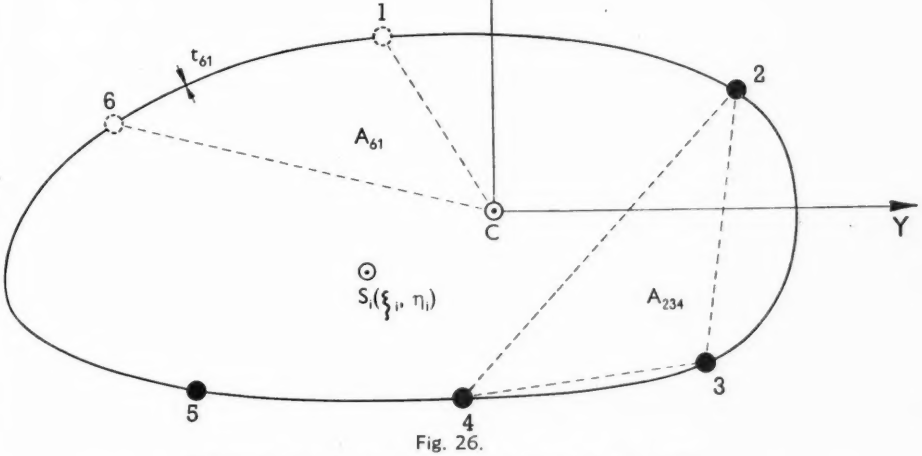


Fig. 26.

The six-boom tube—the formation of the secular equation.

The diagram shows how to form the typical term in the coefficient of $-\lambda^2$ in the secular equation for the six-boom tube. Here n=6 and m=1, so that there will be 6−1−3=2 B's in the expressions of the form (400). There are in all $\binom{6}{2} = 15$ such terms in the coefficient of $-\lambda^2$. Selecting the typical factor $\frac{1}{B_1 B_6}$ one obtains the associated expression (401) by considering the four-boom tube 2, 3, 4, 5 (indicated by full black circles). Thus the whole term takes the form given in (406) and becomes,

$$\frac{1}{B_1 B_6}\{\delta_{52}\,|23,\ 34,\ 45|^2 + \delta_{23}\,|34,\ 45,\ 52|^2 + \delta_{34}\,|45,\ 52,\ 23|^2 + \delta_{45}\,|52,\ 23,\ 34|^2\}$$

5.7.2. *The proof for the secular equation* (397) *for an n-boom tube.*

Equation (397) may be proved directly in the following manner. Select at random $n-m-3$ booms

$$B_r,\ B_s,\ \ldots\ldots\ B_t$$

where $r<s<\ldots\ldots<t$. In the secular determinant (68) add consecutively the rth, ... sth and rth rows and columns to the immediately preceding row and column, starting with the rth. It will then be seen that the booms B_r, B_s, ..., B_t appear only in the rth, sth, ..., and tth diagonal elements. The factor of

$$(-1)^{n-3-m}\frac{1}{B_rB_s\ldots\ldots B_t} \qquad (400a)$$

is the determinant formed by excluding the rth, sth, ..., and tth rows and columns from the *new* determinant and putting all the $\frac{1}{B}$'s equal to zero. This latter determinant will have in the first $(m+3)$ rows and columns only diagonal elements of the form $\lambda^2\delta_{ij}$ where $i, j \neq r, s, \ldots\ldots, t$. Expanding this determinant by the Laplace method using the last three columns and rows one obtains,

$$-(\lambda^2)^m \sum_{\binom{m+3}{3}} [\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\,|ab,\ cd,\ ef|^2]$$

which is identical with the corresponding term in (397).

Equation (397) may also be proved by induction. Assuming the expression (397) to be true for a tube with $(n-1)$ booms it will be shown that it is true also for the n-boom tube. Consider the determinant of equation (68) for an n-boom tube and denote it by Δ_n (see p. 225 of 3.2.2).

The coefficient of the term in $(\lambda^2)^{n-3}$ does not involve any B's and may, by the Laplace expansion using the last three columns and rows, be written,

$$-\sum [\delta_{i,\,i+1}\delta_{k,\,k+1}\ldots\ldots\delta_{p,\,p+1}\,|a\cdot a+1,\ c\cdot c+1,\ e\cdot e+1|^2]$$

which proves expression (405).

Adding the nth row and column to the $(n-1)$th row and column, respectively, one obtains a determinant in which $\frac{1}{B_n}$ occurs only in the nth diagonal element. The determinant now formed by omitting the nth row and column is evidently the determinant Δ_{n-1} for the tube obtained by omitting the boom B_n. Thus in the expansion of Δ_n all the terms involving $\frac{1}{B_n}$ are included in the expression

$$-\frac{1}{B_n}\Delta_{n-1}$$

The secular equation for the n-boom tube may therefore be written,

$$\Delta_n = -(\lambda^2)^{n-3}\sum_{\binom{n}{3}} [\delta_{i,\,i+1}\delta_{s,\,k+1}\ldots\ldots\delta_{p,\,p+1}\,|a\cdot a+1,\ c\cdot c+1,\ e\cdot e+1|^2]$$

$$-\frac{1}{B_n}\Delta_{n-1}+\text{terms in lower powers of }\lambda^2\text{ not involving }\frac{1}{B_n}=0 \qquad (408)$$

The expression Δ_{n-1} is identical with the L.H.S. of equation (397) written for the $(n-1)$ boom tube. It follows that the coefficient of $(-1)^n(\lambda^2)^m$ in equation (408) contains the typical term,

$$(-1)^m\frac{1}{B_nB_s\ldots\ldots B_t}\sum_{\binom{m+3}{3}} [\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\,|ab,\ cd,\ ef|^2] \qquad (409)$$

Since the choice of B_n is arbitrary the typical term (409) is seen to be identical with the corresponding term in the coefficient of $(-1)^{n-1}(\lambda^2)^m$ in the secular equation for the n-boom tube. Hence if equation (397) is true for the $(n-1)$ boom tube it is true also for the n, $n+1$ ..... boom tubes. But it is easily seen to be valid for the four-boom tube (see equations (188) and (189) of 4.3.1) and thus appplies for any single-cell n-boom tube.

5.7.3. *The h-function and the co-ordinates ξ, η for a single-cell n-boom tube.*

To each of the $(n-3)$ roots λ_i of the secular equation (397) will correspond a set of h-functions and ξ, η values. Denoting the co-factors of the Rth element in the $(n+1)$th row of the determinant of equation (68) by Ω_R, one has,

$$\frac{h_{12}}{\Omega_1} = \frac{h_{23}}{\Omega_2} = \ldots\ldots = \frac{h_{R,\,R+1}}{\Omega_R} = \ldots\ldots = \frac{h_{n1}}{\Omega_n}$$

$$= -\frac{\lambda^2}{A}\frac{\sum_1^n \delta_{r,\,r+1} h_{r,\,r+1}}{\Omega_{n+1}} = -\frac{\lambda^2}{2A}\frac{\eta \sum_1^n \delta_{r,\,r+1} h_{r,\,r+1}}{\Omega_{n+2}} = +\frac{\lambda^2}{2A}\frac{\xi \sum_1^n \delta_{r,\,r+1} h_{r,\,r+1}}{\Omega_{n+3}} = \text{const.} \quad (410)$$

The problem therefore reduces to the evaluation of the co-factors. Considering first the co-factors Ω_1 to Ω_n and observing that the determinant Δ_n is a quadratic form in the A's one obtains immediately,

$$\Omega_R = \frac{1}{2}\frac{\partial \Delta_n}{\partial A_{R,\,R+1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (411)$$

Hence differentiating equation (397) *w.r.t* $A_{R,\,R+1}$ and noting that,

$$\left.\begin{aligned} &\frac{\partial A_{ab}}{\partial A_{R,\,R+1}} = 1, \text{ if } R,\ R+1 \text{ is within the circuit } ab \\ \text{and} \\ &\frac{\partial A_{ab}}{\partial A_{R,\,R+1}} = 0, \text{ if } R,\ R+1 \text{ is not within the circuit } ab \end{aligned}\right\} \quad . \quad . \quad (412)$$

one obtains, with the notation,

$$|cd,\ ef| = \begin{vmatrix} x_{cd} & y_{cd} \\ x_{ef} & y_{ef} \end{vmatrix} \quad . \quad . \quad . \quad . \quad (413)$$

$$\Omega_R = (-1)^n \sum_{m=0}^{n-3} (-1)^m (\lambda^2)^m \sum_{\binom{n}{m+3}} \left\{ \frac{1}{B_r B_s \ldots\ldots B_t} \sum_{\binom{m+2}{2}} [\delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq}\, |ab,\ cd,\ ef| \,.\, |cd,\ ef|] \right\} \quad (414)$$

where the determinants $|ab,\ cd,\ ef|$ include only those involving $A_{R,\,R+1}$ and in which the terms (ab) containing $A_{R,\,R+1}$ is *always* written first. Note that the m walls $ij,\ kl,\ \ldots\ldots\ pq$ must not include $R,\ R+1$, and that in all there are $\binom{m+2}{2}$ terms of the form

$$\delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq} |ab,\ cd,\ ef| \,.\, |cd,\ ef| \quad . \quad . \quad . \quad . \quad (415)$$

for each factor,

$$\frac{1}{B_r B_s \ldots\ldots B_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (400)$$

The total number of terms in $(\lambda^2)^m$ will be,

$$\binom{n}{m+3}\binom{m+2}{2} = \frac{n!}{2\,(m+3)\,m!\,(n-m-3)!} \quad . \quad . \quad . \quad (416)$$

The total number of terms in Ω_R will be,

$$2^{n-3}[n^2 - 5n + 8] - 1 \quad . \quad . \quad . \quad . \quad . \quad (417)$$

From equations (410) and the normalising condition (70) of 3.2.2, p. 226,

$$h_{R,\,R+1} = \frac{\Omega_R}{[\sum_1^n \delta_{r,\,r+1}(\Omega_r)^2]^{1/2}} \quad \text{for } R = 1 \text{ to } n \quad . \quad . \quad . \quad (418)$$

An important relation between the co-factor Ω_{n+1} and the co-factors Ω_1 to Ω_n follows from equations (410). Thus,

$$\Omega_{n+1} = -\frac{\lambda^2}{A}\sum_{1}^{n}\delta_{P,\,P+1}\,\Omega_P \qquad . \quad . \quad . \quad (419)$$

The co-factors Ω_{n+1}, Ω_{n+2}, Ω_{n+3} may be obtained by a method similar to that used to prove directly the secular equation,

$$\Omega_{n+1} = (-1)^{n-1}\lambda^2\sum_{m=0}^{n-3}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+3}}\left\{\frac{1}{B_r B_s \ldots\ldots B_t}\sum_{\binom{m+3}{2}}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\delta_{ab}\,|cd,\,ef|^2]\right\} \quad (420)$$

$$\Omega_{n+2} = (-1)^{n}\lambda^2\sum_{m=0}^{n-3}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+3}}\left\{\frac{1}{B_r B_s \ldots\ldots B_t}\sum_{\binom{m+3}{2}}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\delta_{ab}\,|cd,\,ef|\,.\,|cd,\,ef|_y]\right\}$$

. . . . . (421)

$$\Omega_{n+3} = (-1)^{n-1}\lambda^2\sum_{m=0}^{n-3}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+3}}\left\{\frac{1}{B_r B_s \ldots\ldots B_t}\sum_{\binom{m+3}{2}}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\delta_{ab}\,|cd,\,ef|\,.\,|cd,\,ef|_x]\right\}$$

. . . . . (422)

These expressions differ from the secular equation (397) by the factors $-\lambda^2$, $+\lambda^2$ and $-\lambda^2$ respectively and in the forms of the δ summations. To derive these summations one again considers the tube with $(m+3)$ booms obtained by omitting the $(n-m-3)$ booms B_r, B_s ..... B_t from the original n-boom tube. In this new tube one forms all possible products

$$\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\delta_{ab}$$

of $(m+1)$ δ's selected from the $(m+3)$ walls and multiplies by,

$|cd,\,ef|^2$ in equation (420)

$|cd,\,ef|\,.\,|cd,\,ef|_y$ in equation (421)

and

$|cd,\,ef|\,.\,|cd,\,ef|_x$ in equation (422)

formed from the remaining two walls.

For each combination of the B's there will be $\binom{m+3}{2}$ such terms. The total number of terms in $(\lambda^2)^{m+1}$ will be,

$$\binom{n}{m+3}\binom{m+3}{2} = \frac{n!}{2\,(m+1)!\,(n-m-3)!} \qquad . \quad . \quad . \quad (423)$$

In each co-factor (420) to (422) there will be in all

$$\frac{n\,(n-1)}{2}\,[2^{n-2}-1]\ \text{terms} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (424)$$

Now from equations (410) and (419),

$$\sum_{1}^{n}\delta_{P,\,P+1}\,h_{P,\,P+1} = \frac{\sum_{1}^{n}\delta_{P,\,P+1}\,\Omega_P}{[\sum_{1}^{n}\delta_{P,\,P+1}\,(\Omega_P)^2]^{1/2}} = -\frac{A}{\lambda^2}\,\frac{\Omega_{n+1}}{[\sum_{1}^{n}\delta_{P,\,P+1}\,(\Omega_P)^2]^{1/2}} \qquad . \quad (425)$$

and

$$\left.\begin{aligned}\xi &= -\frac{2\Omega_{n+3}}{\Omega_{n+1}}\\ \eta &= +\frac{2\Omega_{n+2}}{\Omega_{n+1}}\end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (426)$$

In the exceptional case when $\sum_{1}^{n}\delta_{P,\,P+1}\,h_{P,\,P+1} = 0$ one obtains, using equation (425),

$$\left.\begin{aligned} a_x &= \xi \sum_1^n \delta_{p,\,p+1} h_{p,\,p+1} = +\frac{2A}{\lambda^2} \frac{\Omega_{n+3}}{[\sum_1^n \delta_{p,\,p+1}(\Omega_p)^2]^{1/2}} \\ a_y &= \eta \sum_1^n \delta_{p,\,p+1} h_{p,\,p+1} = -\frac{2A}{\lambda^2} \frac{\Omega_{n+2}}{[\sum_1^n \delta_{p,\,p+1}(\Omega_p)^2]^{1/2}} \end{aligned}\right\} \quad . \quad . \quad . \quad (426a)$$

If in addition $a_x = a_y = 0$ the corresponding eigenload represents a pure warping system not giving rise to any deflection or twist.

The H- and L-functions, as given in equations (138) and (340) respectively, are in the present case,

$$\left.\begin{aligned} (H_\sigma)_R &= -\frac{A}{\lambda^2} \frac{\Omega_{n+1}}{[\sum_1^n \delta_{p,\,p+1}(\Omega_p)^2]^{1/2}} \qquad (L_\sigma)_R = -\frac{A}{\lambda^3}\left(\frac{E}{G}\right)^{\frac{1}{2}} \frac{\Omega_R - \Omega_{R-1}}{B_R} \frac{\Omega_{n+1}}{\sum_1^n \delta_{p,\,p+1}(\Omega_p)^2} \\ (H_\tau)_{R,\,R+1} &= -\frac{A}{\lambda^2} \frac{\Omega_{n+1}}{[\sum_1^n \delta_{p,\,p+1}(\Omega_p)^2]^{1/2}} \qquad (L_\tau)_{R,\,R+1} = -\frac{A}{\lambda^2} \frac{\Omega_R}{t_{R,\,R+1}} \frac{\Omega_{n+1}}{\sum_1^n \delta_{p,\,p+1}(\Omega_p)^2} \end{aligned}\right\} \quad (427)$$

5.7.4. *Reduction of a multi-cell n-boom tube to an equivalent single-cell tube.*

The type of n-boom multi-cell tube illustrated in Fig. 16, p. 263, is reducible to an equivalent single-cell tube, by the method used in 4.3.2 for the four-boom tube. The detailed procedure to determine the eigenvalues λ_i and the h_i^o- and h_i^i-functions is as follows.

(a) Find the length thickness ratio $\delta_{r,\,r+1}$ of each equivalent wall from,

$$\delta_{r,\,r+1} = \frac{(\delta^o_{r,\,r+1})(\delta^i_{r,\,r+1})}{\delta^o_{r,\,r+1} + \delta^i_{r,\,r+1}} \quad . \quad . \quad . \quad . \quad . \quad (206a)$$

(b) Find the areas $A^e_{r,\,r+1}$, A^e, and A^m from,

$$A^e_{r,\,r+1} = \frac{(\delta^o_{r,\,r+1})(A^i_{r,\,r+1}) + (\delta^i_{r,\,r+1})(A^o_{r,\,r+1})}{\delta^o_{r,\,r+1} + \delta^i_{r,\,r+1}} \quad . \quad . \quad . \quad (207a)$$

$$A^e = \sum_1^n A^e_{r,\,r+1} \quad . \quad . \quad . \quad . \quad . \quad (207b)$$

and,

$$(A^m)^2 = \sum_1^n \frac{(A^{oi}_{r,\,r+1})^2}{\delta^o_{r,\,r+1} + \delta^i_{r,\,r+1}} \quad . \quad . \quad . \quad . \quad . \quad (210a)$$

(c) Find the value of K from,

$$K = \frac{A^e - [(A^e)^2 + (A^m)^2 \sum_1^n \delta_{r,\,r+1}]^{1/2}}{\sum_1^n \delta_{r,\,r+1}} \quad . \quad . \quad . \quad . \quad (214a)$$

(d) Find the equivalent areas $A_{r,\,r+1}$ and A from,

$$A_{r,\,r+1} = A^e_{r,\,r+1} - K\delta_{r,\,r+1} \quad . \quad . \quad . \quad . \quad . \quad (212b)$$

and,

$$A = \sum_1^n A_{r,\,r+1} = A^e - K \sum_1^n \delta_{r,\,r+1} \quad . \quad . \quad . \quad . \quad . \quad (212c)$$

(e) Find λ_i^2, the h-functions, the boundary integral $\Sigma\, \delta_{r,\,r+1}\, h_{r,\,r+1}$ and the ξ, η values for the equivalent single-cell tube from the analysis given in 5.7.1.-3.

(f) Find the $h'_{r,\,r+1}$ and finally $h^o_{r,\,r+1}$ and $h^i_{r,\,r+1}$ from,

$$h'_{r,\,r+1} = h_{r,\,r+1} + \frac{K}{A} \sum_1^n \delta_{r,\,r+1}\, h_{r,\,r+1} \quad . \quad . \quad . \quad . \quad (211a)$$

$$\left.\begin{aligned} h'_{r,r+1} &= \frac{\delta^{i}_{r,r+1}}{\delta^{o}_{r,r+1}+\delta^{i}_{r,r+1}}\,h'_{r,r+1} + \frac{(A^{oi}_{r,r+1}/A)}{\delta^{o}_{r,r+1}+\delta^{i}_{r,r+1}}\sum_{1}^{n}\delta_{r,r+1}\,h_{r,r+1} \\ h^{i}_{r,r+1} &= \frac{\delta^{o}_{r,r+1}}{\delta^{o}_{r,r+1}+\delta^{i}_{r,r+1}}\,h'_{r,r+1} - \frac{(A^{oi}_{r,r+1}/A)}{\delta^{o}_{r,r+1}+\delta^{i}_{r,r+1}}\sum_{1}^{n}\delta_{r,r+1}\,h_{r,r+1} \end{aligned}\right\} \quad (204a)$$

Note that the λ_i^2, $\sum_1^n \delta^{o}_{r,r+1}\,h^{o}_{r,r+1}$ and ξ, η for the actual tube are the same as the corresponding values for the equivalent tube calculated under (e).

5.7.5. *The multi-boom tube with single symmetry.*

When a multi-boom tube possesses an axis of symmetry very considerable simplification of the equations developed in 5.7.1 to 5.7.3, with a corresponding reduction in the numerical work required in a particular case, is possible. Attention will be confined to single-cell tubes but the method of 5.7.4 readily allows extension to multi-cell tubes of the type shown in Fig. 16.

A tube symmetrical about the Y-axis with $2n$ booms is illustrated in Fig. 27. No booms need be placed at α, β on the axis of symmetry, since in such cases the appropriate results may be obtained by coalescence of the pairs of booms (1 and $2n$) and/or (n and $n+1$), so that $\delta_{2n,1}=\delta_{n,n+1}=0$.

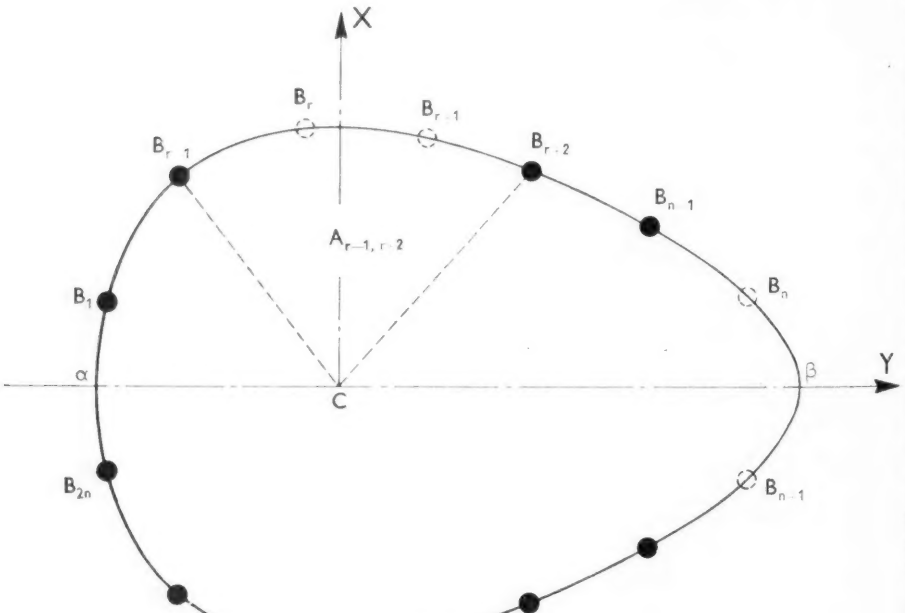


Fig. 27.
The singly symmetrical tube with 2n booms.

Note: When forming the terms under $\frac{1}{B_rB_{r+1}B_n}$ in the secular equation and the co-factors in the b-modes one considers the open tube α, 1, $r-1$, $r+2$, $n-1$, β.

Note that,

$$\left.\begin{aligned} B_r &= B_{2n-r+1} \\ \delta_{r,\,r+1} &= \delta_{2n-r,\,2n-r+1},\quad A_{r,\,r+1} = A_{2n-r,\,2n-r+1} \\ x_{r,\,r+1} &= x_{2n-r,\,2n-r+1},\quad y_{r,\,r+1} = -y_{2n-r,\,2n-r+1} \end{aligned}\right\} \quad . \quad . \quad . \quad (428)$$

As discussed in 5.4.12 the symmetry makes possible the separation of the eigenfunctions into two distinct groups, namely:

(a) eigenfunctions involving flexure in the YZ plane, for which,

$$\left.\begin{aligned} h_{r,\,r+1} &= -h_{2n-r,\,2n-r+1} \\ h_{2n,\,1} &= h_{n,\,n+1} = 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (429)$$

and (b) eigenfunctions involving twist and/or flexure in the XZ plane, for which,

$$h_{r,\,r+1} = h_{2n-r,\,2n-r+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (430)$$

The physical reason for relations (429) and (430) may be seen by considering the shear distribution associated with the two types of deformation.

It is possible, using relations (428), to factorise the secular equation (68) into two secular equations corresponding to the two distinct groups of eigenfunctions. However, separate application of equations (429) and (430) in equations (66a) and (67) over only half the cross-section, gives immediately the two distinct secular equations.

(a) The eigenfunctions for flexure in the YZ plane.

Considering first the eigenfunctions for flexure in the YZ plane, one sees that by virtue of equations (428) and (429),

$$\sum_{1}^{2n} A_{r,\,r+1}\, h_{r,\,r+1} = \sum_{1}^{2n} x_{r,\,r+1}\, h_{r,\,r+1} = 0 \quad . \quad . \quad . \quad (431)$$

so that the first two of conditions (67) are automatically satisfied. Thus the secular determinant becomes:—(see following page, equation (432)).

Expanding this determinant by the method used in 5.7.2 for the direct proof of the secular equation (397), one obtains,

$$\Delta_y = (-1)^{n-1} \sum_{m=0}^{n-2} (-1)^m (\lambda^2)^m \sum_{\binom{n}{m+2}} \left\{ \frac{1}{B_r B_s \ldots\ldots B_t} \sum_{m+1} [\delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq} (y_{ab})^2] \right\} = 0 \quad (432a)$$

where $r,\ s,\ \ldots\ldots,\ t \neq i,\ j, \ldots\ldots q,\ a,\ b$ are boom numbers. The equation yields $(n-2)$ eigenvalues λ_i, which correspond to the flexural modes in the plane YZ. The terms,

$$\frac{1}{B_r B_s \ldots\ldots B_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (400)$$

are formed from all possible products of $(n-m-2)$ different B's selected from the n booms. Thus there are in all $\binom{n}{m+2}$ such terms for the mth power of λ^2.

To form the summation

$$\sum_{m+1} [\delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq} (y_{ab})^2] \quad . \quad . \quad . \quad . \quad . \quad (433)$$

under the terms (400), one considers the *open* tube with $m+2$ booms obtained by omitting the $n-m-2$ booms $B_r,\ B_s, \ldots\ldots, B_t$ from the original n-boom open tube $(1,\ n)$, and joining the remaining booms. In the open tube so formed the $(m+1)$ walls

$$ij,\ kl,\ \ldots\ldots,\ pq,\ ab$$

yield the $(m+1)$ products appearing in (433). The total number of terms in $(\lambda^2)^m$ will be

$$\binom{n}{m+2}(m+1) = \frac{n!\,(m+1)}{(n-m-2)!\,(m+2)!} \quad . \quad . \quad . \quad . \quad (434)$$

$$
\Delta_y \equiv
\begin{vmatrix}
\left[\lambda^2\delta_{12} - \dfrac{1}{B_1} - \dfrac{1}{B_2}\right] & \dfrac{1}{B_2} & 0 & 0 & 0 & 0 & 0 & y_{12} \\
\dfrac{1}{B_2} & \left[\lambda^2\delta_{23} - \dfrac{1}{B_2} - \dfrac{1}{B_3}\right] & \dfrac{1}{B_3} & 0 & 0 & 0 & 0 & y_{23} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \dfrac{1}{B_r}\left[\lambda^2\delta_{r,r+1} - \dfrac{1}{B_r} - \dfrac{1}{B_{r+1}}\right] & \dfrac{1}{B_{r+1}} & 0 & 0 & 0 & y_{r,r+1} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & \dfrac{1}{B_{n-2}}\left[\lambda^2\delta_{n-2,n-1} - \dfrac{1}{B_{n-2}} - \dfrac{1}{B_{n-1}}\right]\dfrac{1}{B_{n-1}} & y_{n-2,n-1} \\
0 & 0 & 0 & 0 & \dfrac{1}{B_{n-1}}\left[\lambda^2\delta_{n-1,n} - \dfrac{1}{B_{n-1}} - \dfrac{1}{B_n}\right] & y_{n-1,n} \\
y_{12} & y_{23} & \cdots\cdots y_{r,r+1} \cdots\cdots & y_{n-2,n-1} & y_{n-1,n} & 0 &
\end{vmatrix}
= 0 \qquad (432)
$$

and in the whole secular equation

$$(n-2)\,2^{n-1}+1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (435)$$

Denoting the co-factors of the Rth element in the nth row of the determinant of equation (432) by Ω_R one has,

$$\frac{h_{12}}{\Omega_1}=\frac{h_{23}}{\Omega_2}=\ldots\ldots=\frac{h_R}{\Omega_R}=\ldots\ldots=\frac{h_{n-1,\,n}}{\Omega_{n-1}}=\frac{\lambda^2}{2A}\frac{a_x}{\Omega_n}=\text{const.} \quad . \quad (436)$$

For $R=1$ to $n-1$

$$\Omega_R=\frac{1}{2}\frac{\partial\Delta_y}{\partial y_{R,\,R+1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (437)$$

Hence,

$$\Omega_R=(-1)^{n-1}\sum_{m=0}^{n-2}(-1)^m(\lambda^2)^m\sum_{\kappa}\left\{\frac{1}{B_rB_s\ldots\ldots B_t}\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}y_{ab}\right\} \quad . \quad (438)$$

where

$\kappa=\binom{n}{m+2}-\binom{R}{m+2}-\binom{n-R}{m+2}$, is the total number of terms in $(\lambda^2)^m$

and ab must always include the wall R, $R+1$. The dependence of κ on R is explained by the fact that in forming the terms (400) with $(n-m-2)$ B's the remaining $(m+2)$ boom *open* tube *must* contain the wall R, $R+1$. Note that if $n<m$, $\binom{n}{m}=0$. From equations (436) and the normalising condition (70),

$$h_{R,\,R+1}=\frac{\Omega_R}{[2\sum_1^{n-1}\delta_{P,\,P+1}(\Omega_P)^2]^{1/2}} \text{ for } R=1 \text{ to } (n-1) \quad . \quad . \quad . \quad (439)$$

The co-factor Ω_n may be expanded as before and takes the form,

$$\Omega_n=(-1)^{n-1}\sum_{m=0}^{n-1}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+1}}\frac{1}{B_rB_s\ldots\ldots B_t}\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq} \quad . \quad (440)$$

where the m δ's include all the m walls of the open tube obtained by omitting the $(n-m-1)$ booms B_r, B_s, $\ldots\ldots$ B_t.

Note that the total number of terms in the co-factors Ω_R and Ω_n are respectively,

$$2^n-2^{n-R}-2^R+1, \text{ and } 2^n-1 \quad . \quad . \quad . \quad . \quad (441)$$

From equations (436) and (439),

$$a_x=\frac{2A}{\lambda^2}\frac{\Omega_n}{[2\sum_1^{n-1}\delta_{P,\,P+1}(\Omega_P)^2]^{1/2}} \quad . \quad . \quad . \quad . \quad . \quad (442)$$

(*b*) *The eigenfunctions for twist and flexure in the XZ-plane.*

For these eigenfunctions, by virtue of equations (428) and (430),

$$\sum_1^{2n}y_{r,\,r+1}\,h_{r,\,r+1}=0 \quad . \quad . \quad . \quad . \quad . \quad (443)$$

so that the last of conditions (67) is automatically satisfied. Thus with the notation,

$$\left.\begin{aligned}\delta_{\alpha 1}&=\frac{1}{2}\delta_{2n,\,1}\;;\;A_{\alpha 1}=\frac{1}{2}A_{2n,\,1}\\ \delta_{n\beta}&=\frac{1}{2}\delta_{n,\,n+1}\;;\;A_{n\beta}=\frac{1}{2}A_{n,\,n+1}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (444)$$

$$
\Delta_{xt} = \begin{vmatrix}
\left[\lambda^2\delta_{o1} - \dfrac{1}{B_1}\right]\dfrac{1}{B_1} & 0 & 0 & 0 & 0 & A_{o1} & x_{o1} \\
\dfrac{1}{B_1}\left[\lambda^2\delta_{12} - \dfrac{1}{B_1} - \dfrac{1}{B_2}\right]\dfrac{1}{B_2} & 0 & 0 & 0 & 0 & A_{12} & x_{12} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \dfrac{1}{B_r}\left[\lambda^2\delta_{r+1} - \dfrac{1}{B_r} - \dfrac{1}{B_{r+1}}\right]\dfrac{1}{B_{r+1}} & 0 & 0 & 0 & A_{r,r+1} & x_{r,r+1} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \dfrac{1}{B_{n-1}}\left[\lambda^2\delta_{n-1,n} - \dfrac{1}{B_{n-1}} - \dfrac{1}{B_n}\right]\dfrac{1}{B_n} & 0 & 0 & A_{n-1,n} & x_{n-1,n} \\
0 & 0 & 0 & \dfrac{1}{B_n}\left[\lambda^2\delta_{n\beta} - \dfrac{1}{B_n}\right] & A_{n\beta} & x_{n\beta} & \\
A_{o1} & A_{12} \cdots A_{r,r+1} \cdots A_{n-1,n} & A_{n\beta} & 0 & 0 & & \\
x_{o1} & x_{12} \cdots x_{r,r+1} \cdots x_{n-1,n} & x_{n\beta} & 0 & 0 & &
\end{vmatrix} = 0 \qquad (445)
$$

the secular determinant becomes as on the preceding page. Expanding this determinant by the method used in 5.7.2 for the direct proof of the secular equation (397), one obtains,

$$\Delta_{\alpha t}=(-1)^{n-1}\sum_{m=0}^{n-1}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+1}}\left\{\frac{1}{B_r B_s \ldots\ldots B_t}\sum_{\binom{m+2}{2}}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\,|ab,\,cd|_\alpha{}^2]\right\}=0 \quad (445a)$$

where $r, s \ldots\ldots t$ are boom numbers, and $i, j, \ldots\ldots, q, a, b, c, d$, which are different from $r, s, \ldots\ldots, t$, are boom numbers and the points α, β. The equation yields $(n-1)$ eigenvalues λ_i which correspond to the twisting modes and flexural modes in the plane XZ. These roots, together with the $(n-2)$ roots of case (a), make the necessary total of $2n-3$ roots.

The terms,

$$\frac{1}{B_r B_s \ldots\ldots B_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (400)$$

are formed from all possible products of $(n-m-1)$ different B's selected from the n-booms. Thus there are in all $\binom{n}{m+1}$ such terms for the mth power of λ^2.

To form the summation,

$$\sum_{\binom{m+2}{2}}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\,|ab,\,cd|_\alpha{}^2] \quad . \quad . \quad . \quad . \quad . \quad (446)$$

under the terms (400), one considers the *open* tube α to β with $(m+1)$ intermediate booms obtained by omitting the $(n-m-1)$ booms $B_r, B_s, \ldots\ldots, B_t$ from the original n-boom open tube and *joining the remaining booms and the two ends* α, β. In the open tube so formed the $(m+2)$ walls

$$ij,\ kl,\ \ldots\ldots,\ pq,\ ab,\ cd,$$

yield the $\binom{m+2}{2}$ products appearing in (446). The total number of terms in $(\lambda^2)^m$ will be,

$$\binom{n}{m+1}\binom{m+2}{2}=\frac{n!\,(m+2)}{2\,(n-m-1)!\,m!} \quad . \quad . \quad . \quad . \quad (447)$$

and in the whole secular equation

$$n\,(n+3)\,2^{n-3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (448)$$

Denoting the co-factors of the elements in the $(n+2)$th row of $\Delta_{\alpha t}$ by

$$\Omega_\alpha,\ \Omega_1,\ \ldots\ldots,\ \Omega_R,\ \ldots\ldots,\ \Omega_{n+2}$$

one has,

$$\frac{h_{\alpha 1}}{\Omega_\alpha}=\frac{h_{12}}{\Omega_1}=\ldots\ldots=\frac{h_{R,R+1}}{\Omega_R}=\ldots\ldots=\frac{h_{n\beta}}{\Omega_n}=-\frac{\lambda^2}{A}\frac{2\sum_\alpha^n\delta_{P,P+1}h_{P,P+1}}{\Omega_{n+1}}$$

$$=-\frac{\lambda^2}{A}\frac{\eta\sum_\alpha^n\delta_{P,P+1}h_{P,P+1}}{\Omega_{n+2}}=\text{const.} \quad . \quad . \quad . \quad . \quad (449)$$

By the methods of the previous paragraphs, for $R=\alpha$ to n,

$$\Omega_R=(-1)^{n-1}\sum_{m=0}^{n-1}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+1}}\left\{\frac{1}{B_r B_s \ldots\ldots B_t}\sum_{m+1}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\,|ab,\,cd|_\alpha x_{cd}]\right\} \quad (450)$$

where ab must always include the wall R, $R+1$, but otherwise the rules of formation for equation (450) are similar to those for the secular equation (445a). The total number of terms in the co-factors (450) is,

$$n2^{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (451)$$

From equations (449) and the normalising condition (70),

$$h_{R,\,R+1} = \frac{\Omega_R}{[2\sum_{\alpha}^{n}\delta_{P,\,P+1}(\Omega_P)^2]^{1/2}} \quad \text{for } R=\alpha \text{ to } n \qquad (452)$$

The co-factors Ω_{n+1} and Ω_{n+2} are,

$$\Omega_{n+1} = (-1)^n\lambda^2 \sum_{m=0}^{n-1}(-1)^m(\lambda^2)^m \sum_{\binom{n}{m+1}} \left\{ \frac{1}{B_r B_s \ldots\ldots B_t} \sum_{m+2} [\delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq}\delta_{ab}(x_{cd})^2] \right\} \qquad (453)$$

$$\Omega_{n+2} = (-1)^{n-1}\lambda^2 \sum_{m=0}^{n-1}(-1)^m(\lambda^2)^m \sum_{\binom{n}{m+1}} \left\{ \frac{1}{B_r B_s \ldots\ldots B_t} \sum_{m+2} [\delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq}\delta_{ab}A_{cd}x_{cd}] \right\} \qquad (454)$$

Note that there are $(n-m-1)$ $\frac{1}{B}$'s and $(m+1)$ δ's in each term under $(\lambda^2)^{m+1}$. The total number of terms in either of these co-factors is,

$$(n+2)\,2^{n-1}-1 \qquad (455)$$

Having Ω_{n+1} and Ω_{n+2} one may write,

$$2\sum_{\alpha}^{n}\delta_{P,\,P+1}h_{P,\,P+1} = \frac{2\sum_{\alpha}^{n}\delta_{P,\,P+1}\Omega_P}{[2\sum_{\alpha}^{n}\delta_{P,\,P+1}(\Omega_P)^2]^{1/2}} = -\frac{A}{\lambda^2}\frac{\Omega_{n+1}}{[2\sum_{\alpha}^{n}\delta_{P,\,P+1}(\Omega_P)^2]} \qquad (456)$$

and

$$\eta = \frac{2\Omega_{n+2}}{\Omega_{n+1}} \qquad (457)$$

In the exceptional case when $\sum_{\alpha}^{n}\delta_{P,\,P+1}h_{P,\,P+1}=0$ one obtains,

$$a_y = \eta 2\sum_{\alpha}^{n}\delta_{P,\,P+1}h_{P,\,P+1} = -\frac{2A}{\lambda^2}\frac{\Omega_{n+2}}{[2\sum_{\alpha}^{n}\delta_{P,\,P+1}(\Omega_P)^2]^{1/2}} \qquad (457a)$$

If, in addition, $a_y=0$ the eigenfunctions represent a warping system not giving rise to any twist or deflection.

5.7.6. *The multi-boom tube with double symmetry.*

Consider a doubly symmetrical tube with $4n$-booms as illustrated in Fig. 28. No booms need be placed on the axes of symmetry at α, γ, etc. since in such cases the appropriate results may be obtained by coalescence of the pairs of booms immediately adjacent to the axes of symmetry.

Because of the double symmetry four distinct groups of eigenfunctions exist. There are:—

(a) Pure flexural modes in the YZ-plane with $(n-1)$ eigenvalues.

(b) Pure flexural modes in the XZ-plane with $(n-1)$ eigenvalues.

(c) Pure twisting modes about the centre of symmetry with n eigenvalues.

(d) Pure warping modes with $(n-1)$ eigenvalues. These modes involve neither flexure nor twist.

Thus there are in all $4n-3$ eigenvalues as required by the general theory. Only the necessary formulæ will be given, since the detailed analysis follows closely the developments of the previous section. Note that, as the cross-section is doubly symmetrical, only a quadrant of the tube need be considered.

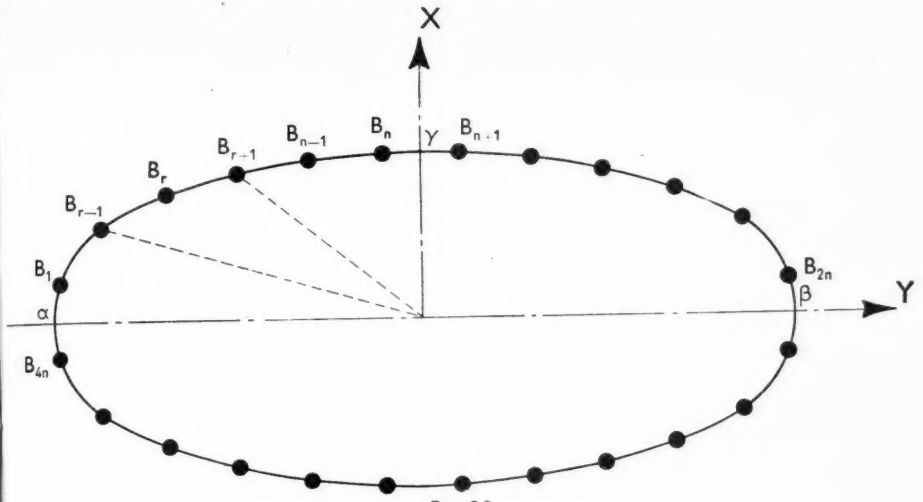


Fig. 28.
The doubly symmetrical tube with 4n booms.

(a) *Pure flexural modes in the YZ-plane.*

The secular equation, noting that $h_{a_1}=0$, is

$$\Delta_y=(-1)^n\sum_{m=0}^{n-1}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+1}}\left\{\frac{1}{B_rB_s\ldots\ldots B_t}\sum_{m+1}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}(y_{ab})^2]\right\}=0 \quad (458)$$

where $r, s, \ldots\ldots, t$ are boom numbers, and $i, j, \ldots\ldots q, a, b$, which are different from $r, s, \ldots\ldots, t$ are boom numbers and the point γ.

To form the summation,

$$\sum_{m+1}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}(y_{ab})^2] \quad . \quad . \quad . \quad . \quad . \quad (459)$$

one considers the *open* tube 1 to γ with $(m+1)$ intermediate booms obtained by omitting the $(n-m-1)$ booms $B_r, B_s, \ldots\ldots, B_t$ from the original n-boom open tube and *joining the remaining booms and the point* γ. In the open tube so formed the $(m+1)$ walls

$$ij, kl, \ldots\ldots, pq, ab$$

yield the $(m+1)$ products appearing in (458).

The h-functions are given by,

$$h_{R,\,R+1}=\frac{\Omega_R}{2[\sum_1^n\delta_{P,\,P+1}(\Omega_P)^2]^{1/2}} \text{ for } R=1 \text{ to } n \quad . \quad . \quad . \quad (460)$$

where,

$$\Omega_R=(-1)^n\sum_{m=0}^{n-1}(-1)^m(\lambda^2)^m\sum_{\kappa}\left\{\frac{1}{B_rB_s\ldots\ldots B_t}\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}y_{ab}\right\} \quad . \quad . \quad (461)$$

where

$$\kappa=\binom{n}{m+1}-\binom{n-R}{m+1}, \text{ is the total number of terms in } (\lambda^2)^m$$

and ab must always include R, $R+1$. The dependence of κ on R is explained by the fact that in forming the terms (400) with $(n-m-1)$ B's the combinations are excluded which contain B_1 to B_R inclusive.

The value of a_x is obtained from,

$$a_x = \frac{A}{\lambda^2} \frac{\Omega_{n+1}}{[\sum_{1}^{n} \delta_{P,\,P+1} (\Omega_P)^2]^{1/2}} \quad . \quad . \quad . \quad . \quad . \quad (462)$$

where,

$$\Omega_{n+1} = (-1)^n \sum_{m=0}^{n} (-1)^m (\lambda^2)^m \sum_{(m)}^{(n)} \left\{ \frac{1}{B_r B_s \ldots\ldots B_t} \delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq} \right\} \quad . \quad (463)$$

Note that there are $(n-m)\,\frac{1}{B}$ s and $m\delta$'s in each term under $(\lambda^2)^m$.

(b) *Pure flexure modes in the XZ-plane.*

The results for the present case can obviously be obtained from (a) by substituting x for y and considering the open tube α, n instead of 1, γ.

(c) *Pure twisting modes about the centre of symmetry.*

The secular equation, noting that the last two of equilibrium conditions (67) are automatically satisfied, is,

$$\Delta_t = (-1)^{n-1} \sum_{m=0}^{n} (-1)^m (\lambda^2)^m \sum_{(m)}^{(n)} \left\{ \frac{1}{B_r B_s \ldots\ldots B_t} \sum_{m+1} [\delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq} (A_{ab})^2] \right\} = 0 \quad (464)$$

where $r, s, \ldots\ldots, t$, are boom numbers and $i, j, \ldots\ldots, q, a, b$, which are different from $r, s, \ldots\ldots, t$, are boom numbers and the points α, γ.

To form the summation,

$$\sum_{m+1} [\delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq} (A_{ab})^2] \quad . \quad . \quad . \quad . \quad . \quad (465)$$

one considers the *open* tube α to γ with m intermediate booms obtained by omitting the $(n-m)$ booms $B_r, B_s, \ldots\ldots, B_t$ from the original n-boom open tube and *joining the remaining booms and the points* α, γ. In the open tube so formed the $(m+1)$ walls

$$ij,\ kl,\ \ldots\ldots, pq,\ ab$$

yield the $(m+1)$ products appearing in (465).

The h-functions are given by,

$$h_{R,\,R+1} = \frac{\Omega_R}{2[\sum_{\alpha}^{n} \delta_{P,\,P+1} (\Omega_P)^2]^{1/2}} \text{ for } R = \alpha \text{ to } n \quad . \quad . \quad . \quad (466)$$

where,

$$\Omega_R = (-1)^{n-1} \sum_{m=0}^{n} (-1)^m (\lambda^2)^m \sum_{(m)}^{(n)} \left\{ \frac{1}{B_r B_s \ldots\ldots B_t} \delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq} A_{ab} \right\} \quad . \quad (467)$$

and ab must always include R, $R+1$. Otherwise the rules of formation for (467) are the same as for (464).

The boundary integral is given by,

$$\sum_{S} \delta_{P,\,P+1}\, h_{P,\,P+1} = -\frac{A}{\lambda^2} \frac{\Omega_{n+1}}{2[\sum_{\alpha}^{n} \delta_{P,\,P+1} (\Omega_P)^2]^{1/2}} \quad . \quad . \quad . \quad (468)$$

where,

$$\Omega_{n+1} = (-1)^n \lambda^2 \sum_{m=0}^{n} (-1)^m (\lambda^2)^m \sum_{(m)}^{(n)} \left\{ \frac{1}{B_r B_s \ldots\ldots B_t} \delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq}\delta_{ab} \right\} \quad . \quad (469)$$

Note that there are $(n-m)$ $\frac{1}{B}$'s and $(m+1)$ δ's in each term under $(\lambda^2)^{m+1}$.

(d) *Pure warping modes.*

The secular equation, noting that $h_{\alpha 1}=h_{n\gamma}=0$, and that all equilibrium conditions (67) are automatically satisfied, is

$$\Delta_w=(-1)^{n-1}\sum_{m=0}^{n-1}(-1)^m(\lambda^2)^m\sum_{(m+1)}^{n}\left\{\frac{1}{B_rB_s\ldots\ldots B_t}\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\right\}=0 \qquad (470)$$

where $r, s, \ldots\ldots, t \neq i, j, \ldots\ldots, q$ are boom numbers. This equation has exactly the same form as the co-factor Ω_n given in equation (440) for the case of flexural modes in the YZ-direction for a singly symmetrical section.

The h-functions are given by,

$$h_{R,\,R+1}=\frac{\Omega_R}{2\left[\sum_{1}^{n-1}\delta_{P,\,P+1}(\Omega_P)^2\right]^{1/2}} \quad \text{for } R=1 \text{ to } (n-1) \quad . \quad . \quad . \quad (471)$$

where,

for $R=1$,

$$\Omega_1=[\Delta_w]_{2-(n-1)}$$

for $R=2$ to $(n-2)$

$$\Omega_R=\frac{1}{B_1B_2\ldots\ldots B_R}[\Delta_w]_{(R+1)-(n-1)} \quad . \quad . \quad . \quad . \quad (472)$$

and for $R=(n-1)$

$$\Omega_{n-1}=\frac{1}{B_1B_2\ldots\ldots B_R\ldots\ldots B_{n-1}}$$

The expression

$$[\Delta_w]_{(R+1)-(n-1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (473)$$

denotes the secular equation Δ_w for the tube obtained by omitting the booms 1 to R and the corresponding booms in the remaining three quadrants of the doubly symmetrical tube. Note that the eigenloads in this case depend only on the boom areas and the δ's. These eigenfunctions do not enter into the stresses and deflections in a transversely loaded tube.

When a tube contains equal booms evenly spaced in skin of constant thickness (finite stringer combinations for example, see also Argyris 1944-46[15]-[17]) great simplifications become possible. Part 6b will contain detailed treatment of such cases with practical approximations to shorten the numerical work.

5.8. The singly symmetrical trapezoidal tube with direct stress carrying top and bottom panels.

This section will demonstrate the application of the general method given in 3.2 to a case where parts of the structure have direct stress carrying walls. It is evident that such a case could be treated by a limiting procedure from the n-boom analysis of 5.7. However, the direct application of the differential equation (45) is preferable.

The trapezoidal tube to be investigated is shown in Fig. 29. Only the top and bottom panels B_1B_2 carry direct stress, the two side panels (spar webs) being purely shear carrying. The eigenfunctions for loads in the YZ-plane (associated with drag loads) will be excluded from the analysis, but may readily be obtained by an artifice from the results for the remaining eigenfunctions (associated with twist and flexure in the XZ-plane).

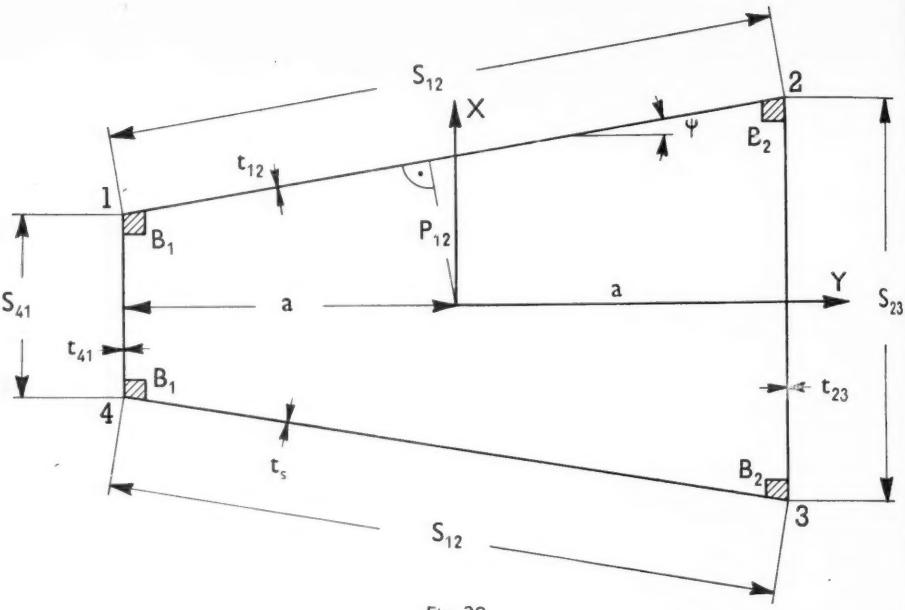


Fig. 29.
Singly symmetrical trapezoidal tube with direct stress carrying top and bottom panels.

5.8.1. *The secular equation.*

With the notation of Fig. 29 the differential equation (45) for the wall (12) may be written,

$$\frac{d^2h_{12}}{ds^2}+\lambda^2\left(\frac{t_s}{t_{12}}\right)h_{12}=\frac{\lambda^2 t_s}{2A}(p_{12}+\eta\sin\psi)\oint\frac{h}{t}\,ds \quad . \quad . \quad . \quad (474)$$

Only the eigenfunctions for twist and flexure in the XZ-plane are being considered so that

$$\xi=0$$

and

$$h_{12}=h_{34} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (475)$$

The solution of equation (474) is,

$$h_{12}=C_1\cos\bar{\lambda}s+C_2\sin\bar{\lambda}s+\frac{t_{12}(p_{12}+\eta\sin\psi)}{2A}\oint\frac{h}{t}\,ds \quad . \quad . \quad . \quad (476)$$

where,

$$\bar{\lambda}=\lambda\left(\frac{t_s}{t_{12}}\right)^{1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (477)$$

and s is measured from the centre of the top panel, positive in the direction 12.

In the walls (23) and (41) the h-functions will have the constant values h_{23} and h_{41}. From the boundary conditions of the type (62), p. 222,

$$B_1\frac{1}{t_s}\left[\frac{dh_{12}}{ds}\right]_1=[h_{12}]_1-h_{41} \quad . \quad . \quad . \quad . \quad . \quad (478)$$

$$B_2\frac{1}{t_s}\left[\frac{dh_{12}}{ds}\right]_2=h_{23}-[h_{12}]_2 \quad . \quad . \quad . \quad . \quad . \quad (479)$$

where the suffices 1 and 2 denote the values of h_{12} and its derivative at the points 1 and 2. Substituting h_{12} from (476) in (478) and (479) one obtains,

$$\left[\frac{B_1\bar{\lambda}}{t_s}\sin\frac{\bar{\lambda}s_{12}}{2}-\cos\frac{\bar{\lambda}s_{12}}{2}\right]C_1+\left[\frac{B_1\bar{\lambda}}{t_s}\cos\frac{\bar{\lambda}s_{12}}{2}+\sin\frac{\bar{\lambda}s_{12}}{2}\right]C_2+h_{41}$$
$$=\frac{t_{12}\sin\psi}{2A}\eta\oint\frac{h}{t}\,ds+\frac{t_{12}p_{12}}{2A}\oint\frac{h}{t}\,ds \quad . \quad . \quad . \quad . \quad (478a)$$

and
$$\left[-\frac{B_2\bar{\lambda}}{t_s}\sin\frac{\bar{\lambda}s_{12}}{2}+\cos\frac{\bar{\lambda}s_{12}}{2}\right]C_1+\left[\frac{B_2\bar{\lambda}}{t_s}\cos\frac{\bar{\lambda}s_{12}}{2}+\sin\frac{\bar{\lambda}s_{12}}{2}\right]C_2-h_{23}$$
$$=-\frac{t_{12}\sin\psi}{2A}\eta\oint\frac{h}{t}\,ds-\frac{t_{12}p_{12}}{2A}\oint\frac{h}{t}\,ds \quad . \quad . \quad . \quad . \quad (479a)$$

Applying the equations of the type (65) to the walls (41) and (23),

$$\frac{2}{t_s}\left[\frac{dh_{12}}{ds}\right]_1+\lambda^2\frac{s_{41}}{t_{41}}h_{41}=\frac{\lambda^2}{2A}s_{41}(a+\eta)\oint\frac{h}{t}\,ds \quad . \quad . \quad . \quad (480)$$

and,
$$-\frac{2}{t_s}\left[\frac{dh_{12}}{ds}\right]_2+\lambda^2\frac{s_{23}}{t_{23}}h_{23}=\frac{\lambda^2}{2A}s_{23}(a-\eta)\oint\frac{h}{t}\,ds \quad . \quad . \quad . \quad (481)$$

Again substituting for h_{12} from (476),

$$2\frac{\bar{\lambda}}{t_s}\sin\frac{\bar{\lambda}s_{12}}{2}\,.\,C_1+2\frac{\bar{\lambda}}{t_s}\cos\frac{\bar{\lambda}s_{12}}{2}\,.\,C_2+\lambda^2\delta_{41}h_{41}=\lambda^2\frac{s_{41}}{2A}\eta\oint\frac{h}{t}\,ds+\lambda^2\frac{s_{41}a}{2A}\oint\frac{h}{t}\,ds \quad (480a)$$

and
$$2\frac{\bar{\lambda}}{t_s}\sin\frac{\bar{\lambda}s_{12}}{2}\,.\,C_1-2\frac{\bar{\lambda}}{t_s}\cos\frac{\bar{\lambda}s_{12}}{2}\,.\,C_2+\lambda^2\delta_{23}h_{23}=-\lambda^2\frac{s_{23}}{2A}\eta\oint\frac{h}{t}\,ds+\lambda^2\frac{s_{23}a}{2A}\oint\frac{h}{t}\,ds \quad (481a)$$

From the conditions (67) for zero shear in the X direction, and zero torque,

$$4\frac{\sin\psi}{\bar{\lambda}}\sin\frac{\bar{\lambda}s_{12}}{2}\,.\,C_1-s_{23}h_{23}+s_{41}h_{41}$$
$$=-2\sin^2\psi s_{12}t_{12}\frac{\eta}{2A}\oint\frac{h}{t}\,ds-2\sin\psi p_{12}s_{12}t_{12}\frac{1}{2A}\oint\frac{h}{t}\,ds \quad . \quad . \quad . \quad (482)$$

$$4\frac{p_{12}}{\bar{\lambda}}\sin\frac{\bar{\lambda}s_{12}}{2}\,.\,C_1+s_{23}ah_{23}+s_{41}ah_{41}$$
$$=-2\sin\psi p_{12}s_{12}t_{12}\frac{\eta}{2A}\oint\frac{h}{t}\,ds-2(p_{12})^2s_{12}t_{12}\frac{1}{2A}\oint\frac{h}{t}\,ds \quad . \quad . \quad . \quad (483)$$

Using the relations,

$$p_{12}=\frac{1}{4}(s_{23}+s_{41})\cos\psi=\frac{a}{2s_{12}}(s_{23}+s_{41})$$
and
$$\sin\psi=\frac{s_{23}-s_{41}}{2s_{12}} \quad . \quad . \quad . \quad . \quad (484)$$

in equations (478a), (479a), (480a), (481a), (482) and (483), the secular equation in the unknowns

$$\frac{C_1}{\bar{\lambda}t_{12}}\,,\ \frac{C_2}{\bar{\lambda}t_{12}}\,,\ h_{23}\,,\ h_{41}\,,\ \frac{\eta}{2A}\oint\frac{h}{t}\,ds,\ \frac{a}{2A}\oint\frac{h}{t}\,ds$$

may be written as on the following page. Expanding the determinant of equation (485) one has,

$$-B_1B_2D(\delta_{12})^2\lambda^4\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}+(B_1+B_2)D\delta_{12}\lambda^2\cos\bar{\lambda}s_{12}$$
$$+\{2[B_1(s_{41})^2+B_2(s_{23})^2](s_{23}+s_{41})^2\delta_{12}-(B_1+B_2)D\}\delta_{12}\lambda^2\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}$$
$$+D\bar{\lambda}s_{12}\sin\bar{\lambda}s_{12}-2\{[(s_{23})^2+(s_{41})^2](s_{23}+s_{41})^2\delta_{12}-D\}\cos\bar{\lambda}s_{12}$$
$$+2[(s_{23})^2-(s_{41})^2]^2\delta_{12}\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}+2\{2s_{23}s_{41}(s_{23}+s_{41})^2\delta_{12}-D\}=0 \quad . \quad (485a)$$

$$
\Delta = \begin{vmatrix}
\left[B_1\lambda^2 \sin\frac{\bar{\lambda}s_{12}}{2} - \bar{\lambda}I_{12}\cos\frac{\bar{\lambda}s_{12}}{2}\right] & \left[B_1\lambda^2\cos\frac{\bar{\lambda}s_{12}}{2} + \bar{\lambda}I_{12}\sin\frac{\bar{\lambda}s_{12}}{2}\right] & 0 & 1 & -\frac{(s_{23}-s_{41})}{2\delta_{12}} & -\frac{(s_{23}+s_{41})}{2\delta_{12}} \\
\left[-B_2\lambda^2 \sin\frac{\bar{\lambda}s_{12}}{2} + \bar{\lambda}I_{12}\cos\frac{\bar{\lambda}s_{12}}{2}\right] & \left[B_2\lambda^2\cos\frac{\bar{\lambda}s_{12}}{2} + \bar{\lambda}I_{12}\sin\frac{\bar{\lambda}s_{12}}{2}\right] & -1 & 0 & \frac{(s_{23}-s_{41})}{2\delta_{12}} & \frac{(s_{23}+s_{41})}{2\delta_{12}} \\
2\sin\frac{\bar{\lambda}s_{12}}{2} & 2\cos\frac{\bar{\lambda}s_{12}}{2} & 0 & \delta_{41} & -s_{41} & -s_{41} \\
2\sin\frac{\bar{\lambda}s_{12}}{2} & -2\cos\frac{\bar{\lambda}s_{12}}{2} & \delta_{23} & 0 & s_{23} & -s_{23} \\
\frac{2(s_{23}-s_{41})}{\delta_{12}}\sin\frac{\bar{\lambda}s_{12}}{2} & 0 & -s_{23} & s_{41} & \frac{(s_{23}-s_{41})^2}{2\delta_{12}} & \frac{(s_{23})^2-(s_{41})^2}{2\delta_{12}} \\
\frac{2(s_{23}+s_{41})}{\delta_{12}}\sin\frac{\bar{\lambda}s_{12}}{2} & 0 & s_{23} & s_{41} & \frac{(s_{23})^2-(s_{41})^2}{2\delta_{12}} & \frac{(s_{23}+s_{41})^2}{2\delta_{12}}
\end{vmatrix} = 0 \tag{485}
$$

where,

$$D=\delta_{23}(s_{41})^4+\delta_{41}(s_{23})^4+2\delta_{12}(s_{23})^2(s_{41})^2 \quad . \quad . \quad . \quad . \quad (486)$$

From the transcendental equation (485a) the eigenvalues λ may be calculated. The nature of the eigenvalue spectrum for such transcendental equations will be discussed in Part 6 of the present paper.

5.8.2. *The h-functions and the co-ordinate η of the centre of twist.*

From the secular equation (485) one may write,

$$\frac{C_1}{\Omega_1}=\frac{C_2}{\Omega_2}=\frac{h_{23}}{\Omega_3}=\frac{h_{41}}{\Omega_4}=\frac{\dfrac{\eta}{2A}\oint\dfrac{h}{t}\,ds}{\Omega_5}=\frac{\dfrac{a}{2A}\oint\dfrac{h}{t}\,ds}{\Omega_6}=\text{const.} \quad . \quad (487)$$

where $\Omega_1, \Omega_2, \ldots\ldots \Omega_6$ are proportional to the co-factors of the elements in the last row of the secular determinant (485). Using the relationship expressed by the secular equation, the functions Ω_1 to Ω_6 are as follows:—

$$\Omega_1=\left[(B_1s_{41}+B_2s_{23})\lambda^2\cos\frac{\bar{\lambda}s_{12}}{2}+(s_{41}+s_{23})\frac{\bar{\lambda}s_{12}}{\delta_{12}}\sin\frac{\bar{\lambda}s_{12}}{2}\right]D \quad . \quad (488)$$

$$\Omega_2=-\left[(B_1s_{41}-B_2s_{23})\lambda^2\sin\frac{\bar{\lambda}s_{12}}{2}+(s_{23}-s_{41})\frac{\bar{\lambda}s_{12}}{\delta_{12}}\cos\frac{\bar{\lambda}s_{12}}{2}-2(s_{23}-s_{41})\frac{1}{\delta_{12}}\sin\frac{\bar{\lambda}s_{12}}{2}\right]D \quad . \quad . \quad (489)$$

$$\Omega_3=-2\left\{[B_1(s_{41})^2+B_2(s_{23})^2]\delta_{12}\lambda^2\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}+(s_{23}-s_{41})^2\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}-[(s_{41})^2+(s_{23})^2]\cos\bar{\lambda}s_{12}+2s_{23}s_{41}\right\}(s_{41}+s_{23})(s_{41})^2 \quad . \quad . \quad (490)$$

$$\Omega_4=-2\left\{[B_1(s_{41})^2+B_2(s_{23})^2]\delta_{12}\lambda^2\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}+(s_{23}-s_{41})^2\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}-[(s_{41})^2+(s_{23})^2]\cos\bar{\lambda}s_{12}+2s_{23}s_{41}\right\}(s_{41}+s_{23})(s_{23})^2 \quad . \quad (491)$$

$$\begin{aligned}\Omega_5=&-\{B_1s_{41}[2\delta_{12}s_{23}(s_{41})^2+\delta_{41}(s_{23})^2(s_{41}+2s_{23})-\delta_{23}(s_{41})^3]\\&-B_2s_{23}[2\delta_{12}(s_{23})^2s_{41}+\delta_{23}(s_{41})^2(s_{23}+2s_{41})-\delta_{41}(s_{23})^3]\}\delta_{12}\lambda^2\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}\\&-\{[\delta_{23}(s_{41})^2-\delta_{41}(s_{23})^2](s_{23}+s_{41})+2\delta_{12}s_{23}s_{41}(s_{23}-s_{41})\}(s_{23}+s_{41})\cos\bar{\lambda}s_{12}\\&+[\delta_{23}(s_{41})^2+\delta_{41}(s_{23})^2+2\delta_{12}s_{23}s_{41}][(s_{23})^2-(s_{41})^2]\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}\\&+2[\delta_{23}(s_{41})^3-\delta_{41}(s_{23})^3](s_{23}+s_{41}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (492)\end{aligned}$$

$$\Omega_6=\left\{[B_1(s_{41})^2+B_2(s_{23})^2]\delta_{12}\lambda^2\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}+(s_{23}-s_{41})^2\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}-[(s_{23})^2+(s_{41})^2]\cos\bar{\lambda}s_{12}+2s_{23}s_{41}\right\}[2\delta_{12}s_{23}s_{41}-\delta_{23}(s_{41})^2-\delta_{41}(s_{23})^2] \quad . \quad (493)$$

Using relations (487) in the normalising condition (57) one obtains,

$$\left.\begin{aligned}&C_1=\frac{\Omega_1}{N^{1/2}},\ C_2=\frac{\Omega_2}{N^{1/2}},\ h_{23}=\frac{\Omega_3}{N^{1/2}},\ h_{41}=\frac{\Omega_4}{N^{1/2}}\\\text{and}\quad&\oint\frac{h}{t}ds=\frac{2A}{a}\frac{\Omega_6}{N^{1/2}}\end{aligned}\right| \quad . \quad . \quad (494)$$

where,

$$N=[(\Omega_1)^2+(\Omega_2)^2]\,\delta_{12}+[(\Omega_1)^2-(\Omega_2)^2]\,\delta_{12}\,\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}+\delta_{23}(\Omega_3)^2+\delta_{41}(\Omega_4)^2$$
$$+2\Omega_1[(s_{23}-s_{41})\Omega_5+(s_{23}+s_{41})\Omega_6]\frac{\sin(\bar{\lambda}s_{12}/2)}{\bar{\lambda}s_{12}/2}$$
$$+[(s_{23}-s_{41})\Omega_5+(s_{23}+s_{41})\Omega_6]^2\frac{1}{2\delta_{12}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (495)$$

Equation (476) may now be written,

$$h_{12}=\frac{1}{N^{1/2}}\left[\Omega_1\cos\bar{\lambda}s+\Omega_2\sin\bar{\lambda}s+\Omega_5\frac{s_{23}-s_{41}}{2\delta_{12}}+\Omega_6\frac{s_{23}+s_{41}}{2\delta_{12}}\right] \quad . \quad . \quad (476a)$$

From equations (476a), (494) and (495) the functions H_i and L_i, as defined in equations (138) and (340), may be calculated. For example the H-functions are,

for the top cover,

$$\left.\begin{aligned}H_\sigma&=\frac{1}{t_8}\left(\frac{Et_8}{Gt_{12}}\right)^{\frac{1}{2}}\frac{2(s_{23}+s_{41})}{N}\Omega_6[-\Omega_1\sin\bar{\lambda}s+\Omega_2\cos\bar{\lambda}s]\\H_\tau&=\frac{1}{t_{12}}\frac{2(s_{23}+s_{41})}{N}\Omega_6\left[\Omega_1\cos\bar{\lambda}s+\Omega_2\sin\bar{\lambda}s+\frac{s_{23}-s_{41}}{2\delta_{12}}\Omega_5+\frac{s_{23}+s_{41}}{2\delta_{12}}\Omega_6\right]\end{aligned}\right\} \quad (496)$$

for the side panels,

$$\left.\begin{aligned}(H_\tau)_{23}&=\frac{1}{t_{23}}\frac{2(s_{23}+s_{41})}{N}\Omega_6\Omega_3\\(H_\tau)_{41}&=\frac{1}{t_{41}}\frac{2(s_{23}+s_{41})}{N}\Omega_6\Omega_4\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (496a)$$

Note that for the bottom cover the H_σ functions have the opposite sign and the H_τ functions have the same sign as in the top cover.

The centres of twist S_i are given by,

$$\eta=a\frac{\Omega_5}{\Omega_6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (497)$$

In the exceptional case when $\Omega_6=0$ one uses in place of (497) the relation,

$$a_y=\frac{2A\Omega_5}{N^{1/2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (497a)$$

If in addition a_y is also zero the corresponding eigenload represents a pure warping system.

5.8.3. *The h-functions and the value of a_x for flexure in the YZ-plane.*

As already stated the eigenfunctions associated with loads in the YZ-plane (drag loads) may be obtained by an artifice from the results of 5.8.1 and 5.8.2. For if $t_{23}=t_{41}=0$ the previous analysis gives the eigenloads corresponding to differential bending of the top and bottom panel. But apart from the difference in sign of the h-functions in the top and bottom panels, the eigenloads associated with drag are identical with those associated with differential bending.

The secular equation is obtained from equation (485a) by dividing by D and taking $t_{23}=t_{41}=0$. Thus one obtains,

$$-[B_1B_2\delta_{12}\lambda^2+(B_1+B_2)]\,\delta_{12}\lambda^2\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}+(B_1+B_2)\,\delta_{12}\lambda^2\cos\bar{\lambda}s_{12}+\bar{\lambda}s_{12}\sin\bar{\lambda}s_{12}+2\cos\bar{\lambda}s_{12}-2=0$$
$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (498)$$

The differential equation for h_{12} is,

$$\frac{d^2h_{12}}{ds^2}+\lambda^2\left(\frac{t_s}{t_{12}}\right)h_{12}=-\frac{\lambda^2 t_s}{2A}\cos\psi\, a_x \quad . \quad . \quad . \quad . \quad (499)$$

the solution of which is,

$$h_{12}=C_1\cos\bar{\lambda}s+C_2\sin\bar{\lambda}s-\frac{t_{12}}{2A}\cos\psi\, a_x \quad . \quad . \quad . \quad . \quad (500)$$

Substituting equation (500) into the zero shear condition in the Y-direction,

$$a_x=(s_{41}+s_{23})\,\delta_{12}\,\frac{\sin(\bar{\lambda}s_{12}/2)}{\bar{\lambda}s_{12}/2}C_1 \quad . \quad . \quad . \quad . \quad . \quad (501)$$

The coefficients C_1 and C_2 can be written,

$$C_1=\frac{\Omega_1}{N^{1/2}}\ ,\ C_2=\frac{\Omega_2}{N^{1/2}} \quad . \quad . \quad . \quad . \quad . \quad (502)$$

Now from equations (488), (489) and (501) it follows that it is possible to put,

$$\Omega_1=\left[(B_1s_{41}+B_2s_{23})\,\lambda^2\cos\frac{\bar{\lambda}s_{12}}{2}+(s_{41}+s_{23})\frac{\bar{\lambda}s_{12}}{\delta_{12}}\sin\frac{\bar{\lambda}s_{12}}{2}\right] \quad . \quad . \quad (503)$$

and,

$$\Omega_2=-\left[(B_1s_{41}-B_2s_{23})\,\lambda^2\sin\frac{\bar{\lambda}s_{12}}{2}+(s_{23}-s_{41})\frac{\bar{\lambda}s_{12}}{\delta_{12}}\cos\frac{\bar{\lambda}s_{12}}{2}-2\,(s_{23}-s_{41})\frac{1}{\delta_{12}}\sin\frac{\bar{\lambda}s_{12}}{2}\right] \quad (504)$$

Then using equation (500) in the normalising condition (57) it follows that:

$$N=[\,(\Omega_1)^2+(\Omega_2)^2]\,\delta_{12}+[\,(\Omega_1)^2-(\Omega_2)^2]\,\delta_{12}\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}-2\,(\Omega_1)^2\,\delta_{12}\left[\frac{\sin(\bar{\lambda}s_{12}/2)}{\bar{\lambda}s_{12}/2}\right] \quad . \quad (505)$$

Equation (500) may also be written,

$$h_{12}=\frac{1}{N^{1/2}}\left[\Omega_1\cos\bar{\lambda}s+\Omega_2\sin\bar{\lambda}s-\Omega_1\frac{\sin(\bar{\lambda}s_{12}/2)}{\bar{\lambda}s_{12}/2}\right] \quad . \quad . \quad . \quad (500a)$$

In equation (339) for the stresses one requires the expressions a_xL_σ and a_xL_τ. They are, for the top cover,

$$\left.\begin{aligned}a_xL_\sigma&=\frac{1}{t_s}\left(\frac{Et_s}{Gt_{12}}\right)^{\frac{1}{2}}\frac{s_{23}+s_{41}}{N}\,\delta_{12}\,\frac{\sin(\bar{\lambda}s_{12}/2)}{\bar{\lambda}s_{12}/2}\,\Omega_1\,[-\Omega_1\sin\bar{\lambda}s+\Omega_2\cos\bar{\lambda}s]\\ a_xL_\tau&=\frac{1}{t_{12}}\,\frac{s_{23}+s_{41}}{N}\,\delta_{12}\,\frac{\sin(\bar{\lambda}s_{12}/2)}{\bar{\lambda}s_{12}/2}\,\Omega_1\left[\Omega_1\cos\lambda s+\Omega_2\sin\bar{\lambda}s-\Omega_1\frac{\sin(\bar{\lambda}s_{12}/2)}{\bar{\lambda}s_{12}/2}\right]\end{aligned}\right\} \quad (506)$$

For the bottom cover L_σ has the same sign, and L_τ the opposite sign to the above.

5.8.4. *The singly symmetrical trapezoidal tube with walls effective only in shear.*

This case may be obtained as a limiting case for $t_s=0$ from the analysis of 5.8.1 and 5.8.2, and serves as a useful check on those results. It is, of course, more easily obtained from the formulæ for the four-boom tube given in 4.3.1. By either method one finds,

for the eigenvalue λ,

$$\lambda^2=\frac{2\,[B_1\,(s_{41})^2+B_2\,(s_{23})^2]\,(s_{23}+s_{41})^2}{B_1B_2D} \quad . \quad . \quad . \quad . \quad (507)$$

for the h-functions,

$$\left.\begin{aligned}h_{12}=h_{34}&=\frac{s_{23}s_{41}}{D^{1/2}}\\ h_{23}=-\frac{(s_{41})^2}{D^{1/2}}\ ,\ h_{41}&=-\frac{(s_{23})^2}{D^{1/2}}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (508)$$

and

$$\sum_{1}^{4} \delta_{12} h_{12} = \frac{2\delta_{12} s_{23} s_{41} - \delta_{23} (s_{41})^2 - \delta_{41} (s_{23})^2}{D^{1/2}} \quad . \quad . \quad . \quad . \quad . \quad (509)$$

where D is given by equations (486). From equations (508), (509) and (62) the H- and L-functions as defined in equations (138) and (340) may be obtained. The co-ordinate η and the value of a_y in the exceptional case are found to be as given in equations (392) and (393) of 5.6.2.

5.8.5. *The doubly symmetrical rectangular tube with direct stress carrying top and bottom panels.*

In this case the secular equation (485a) reduces to,

$$\left[\tan \frac{\bar{\lambda} s_{12}}{2} + \frac{2B}{t_s s_{12}} \frac{\bar{\lambda} s_{12}}{2}\right] \times \left\{\left[\frac{\bar{\lambda} s_{12}}{2} \cot \frac{\bar{\lambda} s_{12}}{2} - \frac{2B}{t_s s_{12}} \left(\frac{\bar{\lambda} s_{12}}{2}\right)^2 - 1\right] 2(\delta_{12} + \delta_{23}) + 8\delta_{12}\right\} = 0 \quad (510)$$

This secular equation yields two distinct sets of eigenvalues.

The first set, for which,

$$\tan \frac{\bar{\lambda} s_{12}}{2} + \frac{2B}{t_s s_{12}} \frac{\bar{\lambda} s_{12}}{2} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (511)$$

corresponds to pure flexural modes in the XZ-plane, the top and bottom panels having equal and opposite end load systems symmetrical about the XZ-plane. Note that these modes are those appropriate to symmetrical diffusion in stringer-sheet panels (see Hildebrand, 1943[14] and Argyris, 1946[17]). It is obvious that, by taking such end load systems with the same sign in the top and bottom panels, one obtains also the pure warping modes analogous to those of 5.7.6(d).

The second set, for which the secular equation may be written,

$$\tan \frac{\bar{\lambda} s_{12}}{2} = \frac{(\delta_{12} + \delta_{23})}{\delta_{23} - 3\delta_{12} + (\delta_{12} + \delta_{23}) \dfrac{2B}{t_s s_{12}} \left(\dfrac{\bar{\lambda} s_{12}}{2}\right)^2} \frac{\bar{\lambda} s_{12}}{2} \quad . \quad . \quad . \quad . \quad (512)$$

corresponds to pure twisting modes.

(*a*) *The flexural modes in the XZ-plane.*

For these modes the secular equation is (511). Either directly or as a special case from the analysis for the trapezoidal tube, one finds,

$$h_{12} = - \frac{\sin \bar{\lambda} s}{\left[\delta_{12} \left(1 - \dfrac{\sin \bar{\lambda} s_{12}}{\bar{\lambda} s_{12}}\right)\right]^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (513)$$

and

$$a_y = -2 s_{12} \left[\frac{\delta_{12}}{1 - \dfrac{\sin \bar{\lambda} s_{12}}{\bar{\lambda} s_{12}}}\right]^{\frac{1}{2}} \frac{\cos (\bar{\lambda} s_{12}/2)}{\bar{\lambda} s_{12}/2} \quad . \quad . \quad . \quad . \quad . \quad (514)$$

The stresses due to transverse loads in the XZ-plane may be written (see the analogous equations (339)).

$$\left.\begin{aligned} \sigma &= \sigma_X + \frac{Q_{x0}}{2A} \sum_i G_z a_{yi} L_{\sigma i} \\ \tau &= \tau_X + \frac{Q_{x0}}{2A} \sum_i G_{ri} a_{yi} L_{\tau i} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (515)$$

Using equations (513) and (514) in (340) one obtains,

$$\left.\begin{aligned} a_y\,(L_y)_{12} &= -\,a_y\,(L_y)_{34} = \frac{2s_{12}}{t_s}\left(\frac{Et_s}{Gt_{12}}\right)^{\frac{1}{2}} \frac{\cos\left(\bar{\lambda}s_{12}/2\right)}{\bar{\lambda}s_{12}/2}\,\frac{\cos\bar{\lambda}s}{1-\dfrac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}} \\ a_y\,(L_r)_{12} &= -\,a_y\,(L_r)_{34} = 2\delta_{12}\,\frac{\cos\left(\bar{\lambda}s_{12}/2\right)}{\bar{\lambda}s_{12}/2}\,\frac{\sin\bar{\lambda}s}{1-\dfrac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}} \end{aligned}\right\} \quad . \quad . \quad (516)$$

where in all cases s is measured from the centre of each panel positive in the clockwise sense.

The G-functions are given in 5.4.

(*b*) *The pure twisting modes.*

For these modes the secular equation is (512). In this case one obtains,

$$\left.\begin{aligned} h_{12} &= \left[\frac{\delta_{12}}{\delta_{12}+\delta_{23}}\right]^{\frac{1}{2}} \frac{1}{N^{1/2}}\left[\frac{\delta_{12}+\delta_{23}}{\delta_{12}}\,\frac{\bar{\lambda}s_{12}/2}{\sin\left(\bar{\lambda}s_{12}/2\right)}\cos\bar{\lambda}s + \frac{\delta_{12}-\delta_{23}}{\delta_{12}}\right] \\ h_{23} &= h_{41} = -\,2\left[\frac{\delta_{12}}{\delta_{12}+\delta_{23}}\right]^{\frac{1}{2}} \frac{1}{N^{1/2}} \end{aligned}\right\} \quad . \quad (517)$$

$$\oint \frac{h}{t}\,ds = +4\,(\delta_{12}-\delta_{23})\left[\frac{\delta_{12}}{\delta_{12}+\delta_{23}}\right]^{\frac{1}{2}} \frac{1}{N^{1/2}} \quad . \quad . \quad . \quad . \quad . \quad (518)$$

where

$$N = 2\,(3\delta_{12}-\delta_{23}) + (\delta_{12}+\delta_{23})\left[\frac{\bar{\lambda}s_{12}/2}{\sin\left(\bar{\lambda}s_{12}/2\right)}\right]^2\left(1+\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}\right) \quad . \quad . \quad (519)$$

The H-functions are,

for the top and bottom panels,

$$\left.\begin{aligned} (H_y)_{12} &= -\,(H_y)_{34} = -\,\frac{4}{t_s}\left(\frac{Et_s}{Gt_{12}}\right)^{\frac{1}{2}} \frac{\delta_{12}-\delta_{23}}{N}\,\frac{\bar{\lambda}s_{12}/2}{\sin\left(\bar{\lambda}s_{12}/2\right)}\sin\bar{\lambda}s \\ (H_r)_{12} &= -\,(H_r)_{34} = \frac{4}{t_{12}}\,\frac{\delta_{12}-\delta_{23}}{N}\left[\frac{\bar{\lambda}s_{12}/2}{\sin\left(\bar{\lambda}s_{12}/2\right)}\cos\bar{\lambda}s + \frac{\delta_{12}-\delta_{23}}{\delta_{12}+\delta_{23}}\right] \end{aligned}\right\} \quad . \quad . \quad (520)$$

where s is again measured as in (a).

For the side panels,

$$H_r = -\,\frac{8}{t_{23}}\,\frac{\delta_{12}-\delta_{23}}{\delta_{12}+\delta_{23}}\,\frac{\delta_{12}}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad (521)$$

(*c*) *The flexural modes in the YZ-plane.*

The secular equation for the present case may be obtained from 5.8.3, or from the preceding section (b) with $t_{23}=0$. In either case the secular equation reduces to,

$$\tan\frac{\bar{\lambda}s_{12}}{2} = \frac{1}{1+\dfrac{2B}{t_s s_{12}}\left(\dfrac{\bar{\lambda}s_{12}}{2}\right)^2}\,\frac{\bar{\lambda}s_{12}}{2} \quad . \quad . \quad . \quad . \quad . \quad (522)$$

This equation gives the modes appropriate to antisymmetrical diffusion in stringer-sheet panels (see Hildebrand 1943[14] and Argyris 1946[17]).

For this case,

$$h_{12}=\frac{1}{[\delta_{12}N]^{\frac{1}{2}}}\frac{\bar{\lambda}s_{12}/2}{\sin(\bar{\lambda}s_{12}/2)}\cos\bar{\lambda}s \quad . \quad . \quad . \quad . \quad . \quad (523)$$

$$a_x=2s_{23}\left[\frac{\delta_{12}}{N}\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (524)$$

where

$$N=-2+\left[\frac{\bar{\lambda}s_{12}/2}{\sin(\bar{\lambda}s_{12}/2)}\right]^2\left(1+\frac{\sin\bar{\lambda}s_{12}}{\bar{\lambda}s_{12}}\right) \quad . \quad . \quad . \quad . \quad (525)$$

For the stresses due to drag loads one requires the expressions,

$$\left.\begin{aligned} a_x(L_\sigma)_{12}&=-a_x(L_\sigma)_{23}=\frac{2s_{23}}{t_s}\left(\frac{Et_s}{Gt_{12}}\right)^{\frac{1}{2}}\frac{\bar{\lambda}s_{12}/2}{\sin(\bar{\lambda}s_{12}/2)}\frac{\sin\bar{\lambda}s}{N}\\ a_x(L_\tau)_{12}&=-a_x(L_\tau)_{34}=\frac{2s_{23}}{t_{12}}\frac{\bar{\lambda}s_{12}/2}{\sin(\bar{\lambda}s_{12}/2)}\frac{\cos\bar{\lambda}s}{N}\end{aligned}\right\} \quad . \quad . \quad (526)$$

Corrigenda to Parts I to V.

In addition to the misprints given in the March issue of the Journal (Monthly Notices) the following misprints occur in Parts I to IV (February 1947 Journal).

p. 220. *6th line from top should read*:—

. . . and (52) for a right cylindrical tube are,

p. 223. *The last expression in equation* (64) *should read*:—

$$\oint_s \frac{h}{t}\,ds$$

p. 245. *The L.H.S. of equation* (146*a*) *should read*:—

$$\frac{d^2g_i}{dz'^2}-\frac{1}{t^*}\frac{dt^*}{dz'}\frac{dg_i}{dz'}-\mu_i{}^2\frac{t^*}{t_s^*}g_i$$

p. 256. *The second line from bottom should read*:—

$$=-4\left[A^*_{34}A-(A^*_{12}A_{34}+A^*_{34}A_{12})+(A^*_{41}A_{23}+A^*_{23}A_{41})\right]^2$$

p. 262. *In Fig.* 15.

s_{12}	should	read	s°_{12}
s_{34}	,,	,,	s°_{34}
A_{12}	,,	,,	A°_{12}
A_{34}	,,	,,	A°_{34}

The following algebraic corrections to paragraphs 4.3.3 and 4.3.8 are important.

p. 263. *Equation* (222) *and that immediately above should read*:—

$$\frac{1}{A^\circ}\left[\sum_1^4\frac{s_{12}}{t_{12}}h'+(A^\circ-A^\circ)C\right]=C$$

or

$$\sum_1^4\frac{s_{12}}{t_{12}}h'-A^\circ C \qquad =0 \quad . \quad . \quad . \quad . \quad . \quad (222)$$

Equation (224) *should read*:—

$$\left.\begin{aligned} h' &= \frac{A^{o}}{A^{B}} \\ C &= \frac{A^{o} \sum_{1}^{4} \frac{s_{12}}{t_{12}}}{A^{B} A^{e}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (224)$$

p. 264. *Equation* (225) *should read*:—

$$A^{B} = \frac{(A^{e})^{2} + (A^{m})^{2} \sum_{1}^{4} \frac{s_{12}}{t_{12}}}{A^{e}} = \frac{A^{2}}{A^{e}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (225)$$

where A is the area of the equivalent tube of 4.3.2.

Equation (226) *should read*:—

$$\left.\begin{aligned} h^{o}{}_{12} &= \frac{A^{o}}{A^{B}} \frac{1}{(s^{o}{}_{12}/t^{o}{}_{12})} \left\{ \frac{s_{12}}{t_{12}} + \frac{A^{o}{}_{12} - A^{e}{}_{12}}{A^{e}} \sum_{1}^{4} \frac{s_{12}}{t_{12}} \right\} \\ h^{i}{}_{12} &= \frac{A^{o}}{A^{B}} \frac{1}{(s^{i}{}_{12}/t^{i}{}_{12})} \left\{ \frac{s_{12}}{t_{12}} + \frac{A^{i}{}_{12} - A^{e}{}_{12}}{A^{e}} \sum_{1}^{4} \frac{s_{12}}{t_{12}} \right\} \end{aligned}\right\} \quad . \quad . \quad . \quad (226)$$

p. 267 and p. 268. *Replace all of paragraph* 4.3.8 *after the* 14*th line from the bottom of p.* 267 *and up to the* 7*th line of p.* 268 *by*:—

$$w_{2}^{*} - w_{1}^{*} = \frac{1}{Gt^{*}} \frac{T}{2A^{o}\rho} \left\{ \frac{s^{o}{}_{12}}{t^{o}{}_{12}} (h_{B})^{o}{}_{12} - \frac{A^{o}{}_{12}}{A^{o}} \sum_{1}^{4} \frac{s^{o}{}_{12}}{t^{o}{}_{12}} (h_{B})^{o}{}_{12} \right\}$$

From the first of equations (226) and with equations (206),

$$w_{2}^{*} - w_{1}^{*} = \frac{1}{Gt^{*}} \frac{T}{2A^{B}\rho} \left\{ \frac{s_{12}}{t_{12}} - \frac{A^{e}{}_{12}}{A^{e}} \sum_{1}^{4} \frac{s_{12}}{t_{12}} \right\} \quad . \quad . \quad . \quad . \quad (247)$$

which can be written

$$w_{2}^{*} - w_{1}^{*} = \frac{1}{Gt^{*}} \frac{TA}{2A^{B}A^{e}\rho} \left\{ \frac{A^{e}}{A} \frac{s_{12}}{t_{12}} - \frac{A^{e}{}_{12}}{A} \sum_{1}^{4} \frac{s_{12}}{t_{12}} \right\} \quad . \quad . \quad . \quad (248)$$

Now using equations (212), (212a), and (225) equation (248) becomes

$$w_{2}^{*} - w_{1}^{*} = \frac{1}{Gt^{*}} \frac{T}{2A\rho} \left\{ \frac{s_{12}}{t_{12}} - \frac{A_{12}}{A} \sum_{1}^{4} \frac{s_{12}}{t_{12}} \right\} \quad . \quad . \quad . \quad . \quad (249)$$

Thus the warping of the actual tube due to the generalised Bredt-Batho system is the same as that of the equivalent tube of 4.3.2. This result follows also from the fact that the equivalent tube of 4.3.2 has the same twist under a given end load system P as the actual tube, and therefore the same warping under a given torque. For a single-cell tube the dimensions in equation (249) are, of course, the true dimensions of the tube.

Now equation (244) may be transformed as follows:—

$$W_{B}^{*} = -2\rho^{2} [(w_{2}^{*} - w_{1}^{*}) A^{*}_{34} - (w_{3}^{*} - w_{2}^{*}) A^{*}_{41} + (w_{4}^{*} - w_{3}^{*}) A^{*}_{12} - (w_{1}^{*} - w_{4}^{*}) A^{*}_{23}] \quad . \quad . \quad . \quad . \quad . \quad (250)$$

Substituting equations (249) in equation (250),

$$W_{B}^{*} = -\frac{T}{AGt^{*}} \rho \left\{ \left[\frac{s_{12}}{t_{12}} A^{*}_{34} - \frac{s_{23}}{t_{23}} A^{*}_{41} + \frac{s_{34}}{t_{34}} A^{*}_{12} - \frac{s_{41}}{t_{41}} A^{*}_{23} \right] - \frac{1}{A} [A^{*}_{34}A_{12} - A^{*}_{41}A_{23} + A^{*}_{12}A_{34} - A^{*}_{23}A_{41}] \sum_{1}^{4} \frac{s_{12}}{t_{12}} \right\} \quad . \quad . \quad . \quad (251)$$

Using equations (190) and (192) in (251), and noting that

$$A^{*}_{12}A^{*}_{34} = A^{*}_{23}A^{*}_{41} \quad . \quad . \quad . \quad . \quad . \quad (252)$$

one obtains with $t^* = \rho^\alpha$,

$$W_B^* = -\frac{T}{AG}\rho^{1-\alpha}\left[\frac{s_{12}}{t_{12}}\bar{A}_{34} - \frac{s_{23}}{t_{23}}\bar{A}_{41} + \frac{s_{34}}{t_{34}}\bar{A}_{12} - \frac{s_{41}}{t_{41}}\bar{A}_{23}\right] \quad . \quad . \quad (253)$$

For a cylindrical tube omit the term $\rho^{1-\alpha}$ from equation (253).

The following misprints have been found in Part V (September 1947 JOURNAL).

p. 770. *The signs of the first terms on the L.H.S. of the first and last equations in* (290) *should be* + *and* − *respectively.*

p. 779. *In the second of equations* (326) *insert after* $\left(\frac{\alpha}{2} + \nu_1\right)$

$$\rho^{\nu_1}$$

The last expressions in wavy brackets in the denominators of equations (327) *should read*:—

$$\left(\frac{\alpha}{2} + \nu_1\right)\rho_e^{-\nu_1} - \left(\frac{\alpha}{2} - \nu_1\right)\rho_e^{\nu_1}$$

PART VI will be published later.

JOINT AERONAUTICAL CONFERENCE

SEPTEMBER 1947 was an outstanding month in the annals of the Society in that the foundations were laid for closer co-operation with The Institute of the Aeronautical Sciences—the Society's counterpart in the United States—by the holding of a joint Aeronautical Conference in London. The Society was host to some 45 prominent members of the Institute and the U.S. Aircraft Industry.

In addition to the full three-day session of lectures at the Conference, and various social functions, which culminated in the dinner at Guildhall on 9th September, the fortnight's intensive activities of the Society finished with the Annual Garden Party on the 14th September. Brief accounts of the Conference, Guildhall Dinner and Garden Party follow.

The first Joint Aeronautical Conference convened by the Royal Aeronautical Society and the Institute of the Aeronautical Sciences was held in London from 3rd to 5th September 1947. It was attended by 362 members of the Society and 45 members of the Institute.

The original suggestion that a Conference should be held was made by the Council of the Society and was welcomed by the Institute, which also suggested that similar meetings should be held at regular intervals alternately in England and the United States.

In addition to the three-day session of lectures a number of social events were arranged for the Conference members, particularly the American visitors, and during the week following the Conference the Americans were the guests of the S.B.A.C. at their Flying Display and Exhibition at Radlett and visited a number of Government establishments and firms in the Industry. The Council of the Society wishes to thank His Majesty's Government, the Ministry of Supply, the S.B.A.C., the Bristol Aeroplane Co. Ltd., the de Havilland Aircraft Co. Ltd., Rolls-Royce Ltd., the Royal Aircraft Establishment, Saunders-Roe Ltd. and the National Gas Turbine Establishment for making such visits possible and for the hospitality extended to the American visitors.

The following was the programme of events during the Conference and the papers presented. The three-day session for the presentation of papers was held at the Victoria Halls, Bloomsbury Square, W.C.1.

Tuesday, 2nd September 1947

A Reception by the President and Council of the Society at 4 Hamilton Place to welcome all delegates to the Conference.

Wednesday, 3rd September 1947, at the Victoria Halls.

MORNING

In the South Hall

9.00 a.m.	Opening remarks by the Presidents of the Royal Aeronautical Society and the Institute of the Aeronautical Sciences.
9.15 a.m.—10.45 a.m.	Paper by Dr. Theodore von Karman on "Theoretical Consideration on High Speed Stability and Control."
11.15 a.m.— 12.45 p.m.	Paper by M. B. Morgan, Royal Aircraft Establishment, on "Control in Low Speed Flight."

In the North Hall

9.15 a.m.—10.45 a.m. Paper by K. T. Spencer, Ministry of Supply and P. A. Hufton of A. & A.E.E., Boscombe Down, on "Modern Operational Factors Affecting Airworthiness."

11.15 a.m.— 12.45 p.m. Paper by Abe Silverstein of the Flight Propulsion Research Laboratory, National Advisory Committee for Aeronautics, on "Icing Problems."

AFTERNOON

In the South Hall

2.15 p.m.— 3.45 p.m. Paper by Dr. Clark B. Millikan, Acting Director of the Daniel Guggenheim Aeronautical Laboratory, California Institute of Technology, on "High Speed Testing in the Southern California Co-operative Wind Tunnel."

4.15 p.m.— 5.45 p.m. Paper by W. G. A. Perring, Director of the Royal Aircraft Establishment, on "High Speed Performance."

In the North Hall

2.15 p.m.— 3.45 p.m. Paper by Grover Loening of the National Advisory Committee for Aeronautics on "The Economics of Personal Aircraft."

4.15 p.m.— 5.45 p.m. Paper by Martin Kiebert, of Sherman Fairchild and Associates, on "Electronic Instrumentation and Control of Aircraft."

Evening

A dinner at 4 Hamilton Place given by the President and Council to the American delegates and their wives.

Thursday, 4th September 1947

MORNING

In the South Hall

9.15 a.m.—10.45 a.m. Paper by H. Knowler, Director and Chief Designer of Saunders-Roe Ltd., on "Structural Problems of Large Aircraft."

11.15 a.m.— 12.45 p.m. Paper by Nicholas J. Hofi, Professor of Aeronautical Engineering, Polytechnic Institute of Brooklyn, on "Thin Walled Monocoques."

In the North Hall

9.15 a.m.—10.45 a.m. Paper by Robert R. Gilruth of the Langley Memorial Aeronautical Laboratory, National Advisory Committee for Aeronautics, on "Résumé and Analysis of N.A.C.A. Wing Flow Tests."

11.15 a.m.— 12.45 p.m. Paper by E. J. Richards, Chief Aerodynamicist, Vickers-Armstrongs Ltd., Weybridge Works, on "Design Problems Arising from Sweepback."

AFTERNOON

In the South Hall

2.15 p.m.— 3.45 p.m. Paper by Dr. D. Williams of the Royal Aircraft Establishment on "Some Novel Structural Properties of Stressed Skin Wings."

4.15 p.m.— 5.45 p.m Paper by A. E. Raymond, Vice-President Engineering, Douglas Aircraft Co. Inc., on "Some Aspects of Transport Aeroplane Development." Read by M. E. Oliveau, Director European Division of Douglas Aircraft Corporation.

In the North Hall

2.15 p.m.— 3.45 p.m. Paper by Robert M. Stanley, Vice-President Engineering, Bell Aircraft Corporation, on "Helicopter Power Plant Installations."

Friday, 5th September 1947

MORNING

In the South Hall

9.15 a.m.—10.45 a.m. Paper by A. G. Elliott, Director and Chief Engineer of Rolls-Royce Ltd., on "Turbine Engine Icing Problems,"

11.15 a.m.— 12.45 p.m. Paper by Ronald M. Hazen, Director—Engineering, Allison Division, General Motors Corporation, on "Factors Affecting Future Development of Reciprocating Engines."

In the North Hall

9.15 a.m.—10.45 a.m. Paper by T. P. Wright, Administrator, C.A.A., on "Personal Aircraft—An American Appraisal."

11.15 a.m.— 12.45 p.m. Paper by R. Hafner, Chief Designer, Helicopter Department, Bristol Aeroplane Co. Ltd., on "Helicopter Rotor Systems and Control Problems."

AFTERNOON

In the South Hall

2.15 p.m.— 3.45 p.m. Paper by F. M. Owner, Chief Engineer (Turbines), Bristol Aeroplane Co. Ltd., on "The Propeller Turbine Aero-Engine."

4.15 p.m.— 5.00 p.m. Concluding remarks by the Presidents of the Society and of the Institute.

Evening

Reception at Lancaster House given by H.M. Government.

Saturday, 6th September 1947

Reading and District Branch Garden Party at Woodley Aerodrome.

Sunday, 7th September 1947

Garden Party at Limes House, Stanmore, Middlesex, given by the President, Sir Frederick Handley Page and Lady Handley Page.

DINNER AT GUILDHALL

Following the Joint Aeronautical Conference, a Dinner at Guildhall was held on Tuesday, 9th September, 1947. Sir Frederick Handley Page, C.B.E., President of the Royal Aeronautical Society, and Lady Handley Page, received the distinguished company.

The guests included His Excellency the American Ambassador, Mr. Lewis Douglas and Mrs. Lewis Douglas, the Lord Mayor and Lady Mayoress of London, the Rt. Hon. Sir Bracewell Smith and Lady Bracewell Smith; and some forty members of the Institute.

By permission of the Air Council, the orchestra of the Central Band of the Royal Air Force provided music during dinner; Grace and the Thanksgiving were rendered by members of St. Paul's Cathedral Choir.

Greetings

The following telegrams were read by Dr. H. Roxbee Cox, D.I.C., F.R.Ae.S., President-Elect.

From Dr. Edward Warner, an Honorary Fellow of both the Royal Aeronautical Society and of the Institute of the Aeronautical Sciences, and President of the International Civil Aviation Organisation:—

> "Congratulations to the Royal Aeronautical Society and the Institute of the Aeronautical Sciences on so notable a series of meetings. It is another concrete manifestation of the power to co-operate. Deeply regret cannot be with you."

From Mr. Lester Gardner, the Founder of the Institute of the Aeronautical Sciences, and for many years its Secretary:—

> "Greetings. Hope conference great success. Regards to all my friends."

From Mr. Griffith Brewer, a Past-President of the Royal Aeronautical Society, a friend of the Wright Brothers and the first Englishman to fly in a heavier-than-air machine:—

> "Please express my warmest appreciation to the Royal Aeronautical Conference for their greetings and my deep regret at being unable to be with them on this historic occasion."

Dr. Roxbee Cox added that Mr. Griffith Brewer had been very ill, but that all his friends were delighted to know that he was getting better and would be with them again in the not too distant future.

The Toasts

Sir Oliver Simmonds, F.R.Ae.S. (Vice-President of the Society) proposed "The Right Hon. The Lord Mayor, Aldermen, Sheriffs and the Corporation of London."

It was an honour on behalf of the great company assembled, to express appreciation to the Lord Mayor on this occasion.

Through his courtesy, the assembled company were seated with the memories of 500 years—the making of Kings, the trials of heritage, the triumphal march of the victor. Edward IV, Cramer, Cromwell, Marlborough, Pitt, Nelson and Wellington were all there; and it was not so long ago since the greatest of all the Honorary Fellows of the Royal Aeronautical Society, Mr. Winston Churchill, had come to Guildhall to hurl defiance at their foes and to bid them, on both sides of the Atlantic, be of good cheer and fight through to victory.

Although the Institute of the Aeronautical Sciences and the Royal Aeronautical Society had worked in the closest harmony throughout the years, the present occasion was the first on which they had held a joint meeting. He trusted sincerely that it would not be long before the second such meeting was held. In this country they had learned to appreciate their American friends more intimately than had been possible in the course of the fleeting visits which each had paid to the shores of the other.

The Conference was over, but was not complete until they had met together as they were at Guildhall. They and their American friends were met together as cousins, as very good friends, to testify to a deep sense of common regard and an unswerving unity of purpose, that a fuller unity which, with steady crescendo, now animated their two great nations. It was the essential foundation of peace and, if all else failed, the guarantee of their common heritage.

The Right Hon. The Lord Mayor: The ancient City of London, founded as it was 2,300 years ago by the Romans, had seen many adventures launched. Their sailors and soldiers, and even their political life, had been guided by that ancient City. In view of the City's history of pioneer work in every branch of defence, of commerce, of industry, of travel, he felt that it was fitting to celebrate the conclusion of the first Anglo-American Aeronautical Conference in Guildhall.

He emphasised the pleasure and pride which the City had in being able to offer Guildhall to the members of the conference on that occasion. In the New Year of 1941 the Hall in which they were met was but a shell; the roof was burned out and the fabric of much of the building was a heap of ruins. The Council Chamber and everything in it were destroyed, and the Hall was the only room left where the Council could work. In it they held the Corporation meetings, ceremonies for the presentation of the Freedom, and the King and Queen were received there on their return from South Africa. Thus, the room was used for every conceivable purpose; but he could not imagine that it could be used for any more useful purpose than for celebrating the meeting between the Royal Aeronautical Society and the Institute of the Aeronautical Sciences. He hoped that in their conference they had reached a certain degree of agreement, that the debates had led to a closer understanding of each other's difficulties and that, whatever ensued from the meetings, the alliance between the United States and Britain would be stronger. The City was proud that they had thought it advisable to celebrate the culmination of the Conference in that ancient building, and on behalf of the City he thanked them.

His Excellency The American Ambassador, Mr. Lewis Douglas: He was privileged to propose the toast of "The Royal Aeronautical Society" not only because of the distinction which the Royal Aeronautical Society had rightfully earned, but also because the function that evening marked the first of many occasions on which the Society and the Institute of the Aeronautical Sciences would meet, alternately in this country and in his.

The present was but a legacy of the past, the events of to-day were but reflections of that which had gone before. That was no less true of the natural sciences and of the applications of scientific knowledge to their way of life than it was of development in the arts and letters and in politics, in the way in which human beings behaved under a certain set of circumstances and conditions. If they were astonished, startled and shocked by the sudden blossoming of nuclear energy, they must go back, within their own time, to Rutherford, and dig farther back into the past to unearth the roots of the tree of their knowledge. If they were to understand the modern aeroplane and the evolution of aero-

dynamics they must not be unmindful of the pioneers who had set forth on their glorious adventure to discover knowledge and the ways and means which now permitted them to demolish time and place.

One of the most notable events in the history of aviation was the establishment, in 1866—81 years ago—of the Royal Aeronautical Society. As notable as its establishment had been the fashion in which it had husbanded, cherished and exchanged the knowledge which it and its members had wrung from a stubborn nature throughout the course of four-fifths of a century; as notable as its establishment was the way in which, true to the tradition of modern science, it had liberally given everything that it had learned. Equally notable was the present occasion, on which, joining with the Institute of the Aeronautical Sciences of his own country, it was freely, unrestrained, unhampered and unfettered, receiving and giving of the knowledge which the members of both bodies had garnered and gathered unto themselves, thus advancing everywhere the cause of science. But more important even than that, the meeting of the two bodies in that unrestrained and unfettered fashion was symbolic of the similarity of interests which bound their two countries together. If those who were actively engaged in the field of politics could have for their guidance the same objectives, if those who were so engaged enjoyed the same criteria which those who had taken part in the Conference enjoyed, they would all be able to view the minor difierences, which sometimes appeared to cloud that deep commonality of interest between them, in their proper perspective.

The fabric of society had been weakened by the devastations and the strain of the two great world wars. It was trite to say that, in troubled times, tensions began to appear; but it was not trite to say that if they could view the superficial tensions in their proper perspective, measure them in the light of the great strain on the world, there could be no doubt about the companionship between this great nation and his own.

And so, in proposing a toast to the Royal Aeronautical Society and to its President, he could do no better than to quote a Professor of Harvard University:—

> "Only a few can increase the patrimony of science, but we can all join in preserving it and in honouring those who have built it."

The President, Sir Frederick Handley Page, C.B.E.: He appreciated the great responsibility he bore in replying to the toast so felicitously proposed by the Ambassador. Members of the Royal Aeronautical Society would wish him at once to thank the American Ambassador a hundredfold for what he had said in regard to the part which the Society had played in this great world of theirs.

To all present it was a wonderful thing to be entertained in Guildhall and to entertain the Ambassador of that country which had seen the birth of the aeroplane as they knew it to-day. But it also appealed to him, standing in that hall, that they of this country and their brothers across the seas had a common heritage, as they saw the monuments to past great men and the unfortunate results of the attacks by an alien autocracy on their common civilisation.

The Royal Aeronautical Society, which was founded in the year 1866, had celebrated its 80th anniversary in Guildhall last year. He had much pride in the fact that he had joined the Society 40 years ago, in the autumn of 1907 when it was the Aeronautical Society of Great Britain and Ireland.

In the Conference which was now ending so happily, they had worked together on common lines and had cemented a friendship which, after all, was underlying all the time and needed only such a conference to bring it to the surface.

He was happy to be able to announce that the Society had been able, through the courtesy of Mr. J. E. Hodgson, its Honorary Librarian, to obtain a very fine collection of old prints, engravings and historical documents relating to aviation, extending back 120 years. The collection had been purchased and presented to the Society and would be one of its most cherished possessions during the years to come. It would be known as the Hodgson-Cuthbert Collection.

During the Conference, they had come down to hard facts and had examined every phase of aeronautical activity.

He hoped that they would devote a great deal of attention to increasing the safety of aeroplanes in all conditions, rather than to thinking too much about how much bigger they could make aircraft.

Perhaps the matter which had held the most interest for everybody in the Conference —at least, it had interested him—was the attack on the supersonic region, into which they might fly, a subject which entailed so much research work and so much development. It was difficult, in a mixed company, to speak on any matter of technical development, but it was interesting to note that, as they came near to the speed of sound, there came about suddenly a tremendous increase of resistance, and if they were going to fly through the speed of sound and go on beyond to 1,000 m.p.h., they must go through what was called a compressibility wall. It was extraordinary that, when they began to fly really fast, they came up against something which was hard, and unless an aeroplane were provided with tremendous power it could not go through.

It was most interesting, to everybody engaged in developing what was new, to peer into the future, to consider whether mankind would be able to travel at speeds in excess of the speed of sound. It would be quite reasonable, in so far as the high speed record was concerned, to anticipate that the stout-hearted people who were willing to hazard their lives in the adventure would be able to get through the compressibility wall and to fly at the very high speeds. He doubted whether the ordinary person would be willing so to do.

Over all the discussions on technical matters, whether concerned with helicopters, aerodynamics, structures, materials or anything else, they had been ascertaining facts, seeking to advance their common knowledge. But he rated as the highest accomplishment of the Conference the renewal of friendships with those from across the Atlantic and the making of many new friends. They had had a marvellous gathering, and it had been a revelation of the immense amount of work they could do together, where there were common problems, common difficulties, common Governmental obstructions occasionally; they felt they could get over the difficulties, in spite of the regulations with which they had to deal.

The Right Hon. The Lord Greenwood, P.C., K.C., proposed "The Institute of the Aeronautical Sciences": In the Guildhall of the City of London, Kings and kingly men had been entertained over the centuries. The members of the Institute had received the greatest token of consideration possible from his Right Honourable Friend, the Lord Mayor, and the Corporation, in supporting the Royal Aeronautical Society in welcoming them there.

It was only a few years ago since men and women were killed within a few yards of where they were sitting, during the many blitzes on London. For hundreds of years the old Guildhall had faced the difficulties that had beset them throughout British history, before

there was a United States. The visitors from the United States, the members of the Institute, were welcomed not only by Britishers who were present on that occasion, but by the whole realm. They all remembered with gratitude, which would last throughout their lives, the coming of the United States into the late conflict; in his opinion, that had saved the world. They all remembered the part played by aviation.

The Institute had between 5,000 and 6,000 members; it was great, and it was growing. The Royal Aeronautical Society had about 6,000 members, and it was still growing. They represented together the brain power in the development of aviation. The impetus of the war had encouraged that development in the most extraordinary way. It could not now be checked, but must go on.

In this country they welcomed the visitors. They had come in the worst period, or one of the worst periods, in history. But although they were distracted and worried, and would be even more worried, the people were determined to carry on, not only in aviation, but in all the ordinary business of life. As Chairman of many big businesses, he could see no sign yet of any weakening in the determination to face the future with as good a heart as that with which they had faced the late disastrous war. But they leaned on the new world, and were bound to do so. As a native-born Canadian, knowing a great deal about Canada and something about the United States, he was profoundly impressed by the fact that there was more good will in the new world towards this old Britain than was usually thought; there had been a complete change in the mental attitude of the American people during and since the war.

He proposed the health of the Institute and wished its members well in their new profession—for it was a new profession—he hoped there would be annual gatherings on one side or other of the Atlantic. Science, like art, knew no frontiers, and there must be the greatest comradeship among those associated with the scientific development of aviation. He coupled with the toast the name of Mr. Preston Bassett, the President of the Institute.

Mr. Preston R. Bassett, responded: The occasion was so impressive that it was difficult to know what to say or where to begin. However, recalling the Lord Mayor's hope that they were agreed on many matters, and Sir Frederick's remark that they had had many controversies, he said they had indeed reached one particular realm of agreement; they were going to meet again, and often.

Giving an account of what he called "the education of an American in one month," he said that exactly a month ago he was in California, where 300 or 400 of the members of the Institute of the Aeronautical Sciences had attended a banquet to celebrate the fifteenth anniversary of the founding of the Institute. With the background of Hollywood, whose traditions were neither deep nor too old, they had really felt a splendid maturity on that occasion.

From California they had come to New York, and then on to London to attend the first joint meeting of the Institute with the Royal Aeronautical Society. A week ago, about 40 of the visiting members of the Institute attended their first dinner with members of the Royal Aeronautical Society, in the Society's home in Hamilton Place. After dinner they were shown the archives of the Society; and the fifteen years of the Institute had paled into insignificance as compared with the 81 years of history of the Royal Aeronautical Society. They had seen some wonderful records and models by pioneers of heavier-than-air flight, such as Cayley, Henson, Stringfellow and Pilcher, and scientists such as Wenham and Glaisher, who had known more about the stratosphere than had anybody since their

time until very recent years. The members of the Institute began to realise that the Society, 81 years of age, was really old. But then to-night, at the end of the Conference, they were invited to Guildhall, which, he was informed, was more than 500 years old.

Fifteen, eighty-one, five hundred—such was the education of this American in one month.

There was a time in the history of Guildhall when it was a meeting place for the Guilds. He could not help feeling that it was really fitting that members of the profession of aeronautics should also meet there on the occasion of the joint conference of the Society and the Institute. Centuries ago men of the skilled trades and professions began to realise that they had a responsibility to each other; they also began to realise that they had a responsibility to the people they served. The visitors to Guildhall on the present occasion felt very young as a profession; yet they were cognisant that they, too, had that responsibility. The profession had grown, in the few years of its existence, to one of tremendous importance; indeed, sometimes they felt that its importance was alarmingly great. But there was no question that their responsibility had grown proportionately, both now and for the future.

One of the surprising and bewildering facts about the profession was the increase in its complication. It had started relatively simply as some of the Guilds; for example, the main responsibility of the Pewter Guild was to control the percentage of lead which could be put into the pewter. But the tree of aeronautical science had grown so many new branches that the aeronautical engineer felt sometimes like a squirrel in a great oak tree. It was because there were so many branches to explore, and more new branches were growing, that the time was most auspicious for the two bodies concerned with aeronautical science to get together. They had all profited very much by their joint meeting, not only in good fellowship, but also in the interchange of ideas. Although there were differences of opinion, there was usually more than one way of solving a problem. Comparisons of the various approaches to the problems had proved most stimulating. It really was time that the two main bodies interested in aeronautics should get together. After all, their nations spoke the same language, and their ideas were not divergent, although there might be temporary variations in those ideas; it might be said that the difference in their trends from time to time was merely a phase difference as between the one country and the other in their pursuit of common objectives.

Therefore, they felt that their joint meeting, the success of which had been even greater than they had anticipated, would be repeated, and would cement their bonds of friendship in the years to come, to their mutual profit. He hoped that within a short time many of their colleagues from this country would go to the United States to attend the next joint meeting. All who had come from America to attend the conference in London wished to thank their British colleagues most sincerely.

The following is a list of members and guests who attended the dinner at Guildhall:—

C. F. Abell, Esq., O.B.E., F.R.Ae.S.; Mrs. Abell; J. Adam, Esq., A.F.R.Ae.S.; Mrs. Adam; J. Aherne-Heron, Esq., A.R.Ae.S.; Mrs. Aherne-Heron; H. J. Allen, Esq.; L. S. Armandias, Esq.; Mrs. Armandias; Alderman and Sheriff Sir George Aylwen; Lady Aylwen.

C. A. Baker, Esq., A.M.I.E.E., M.I.E.I., A.R.Ae.S.; Major C. J. Ball, D.S.O., M.C., F.R.Ae.S.; Air Commodore F. R. Banks, C.B., O.B.E., F.R.Ae.S. (Member of Council of Royal Aeronautical Society); Mrs. Banks; Dr. A. Buchanan Barbour, O.B.E., F.R.Ae.S.; Mrs. Buchanan Barbour; Miss F. E. Barwood; E. L. Bass, Esq., F.R.Ae.S.; Mrs. Bass; P. R. Bassett Esq. (President of the Institute of the Aeronautical Sciences); Miss Baube; D. T. F. W. Baxter, Esq., A.R.Ae.S.; Mrs. Baxter; R. B. Beisel, Esq. (Vice-President of the Institute of the Aeronautical Sciences); J. Bell, Esq., A.R.Ae.S.; Wing Commander

J. C. G. Bell; Mrs. Bell; P. Benham, Esq.; A. Gordon Berg, Esq., F.R.Ae.S.; Mrs. Berg; Dr. K. G. Bergin, M.A., M.D., A.F.R.Ae.S.; Mrs. Bergin; General B. E. Bertrandias; Mrs. Pamela Bewg; H. G. Bloss, Esq., A.F.R.Ae.S.; A. C. Bolton, Esq., A.F.R.Ae.S.; E. W. Bonar, Esq., G.C., A.F.R.Ae.S.; Mrs. Bonar; S/Ldr. T. A. V. Booth, A.R.Ae.S.; Mrs. The Captain T. A. V. (G.T.C.) Booth; Rear-Admiral M. W. S. Boucher, D.S.O., A.F.R.Ae.S.; Mrs. Boucher; L. C. Bowker, Esq.; Mrs. F. D. Bradbrooke, A.R.Ae.S.; Air Commodore H. G. Brackley, C.B.E., D.S.O., D.S.C., A.F.R.Ae.S.; J. W. S. Brancker, Esq., A.R.Ae.S.; L. G. Brazier, Esq., B.Sc., A.C.G.I., A.F.R.Ae.S.; Mrs. Brazier; Leonard Bridgman, Esq.; Lt.-Col. W. A. Bristow, F.Inst.F., M.I.E.E., M.I.M.E., F.R.Ae.S.; Mrs. Bristow; H. W. J. Brittlebank, Esq., B.Sc.(Eng.), A.F.R.Ae.S.; Robert Broadbent, Esq., M.I.Mech.E., M.I.I.A., A.M.I.P.E.; Mrs. Broadbent; Miss M. C. Brock; Professor S. Brodetsky, M.A., Ph.D., F.R.Ae.S.; Mrs. Brodetsky; E. C. Brown, Esq., A.F.R.Ae.S.; Mrs. Brown; P. Brown, Esq.; Sir John S. Buchanan, C.B.E., F.R.Ae.S. (Member of Council of Royal Aeronautical Society); Lady Buchanan; F. E. Buckell, Esq., A.F.R.Ae.S.; Major G. P. Bulman, C.B.E., B.Sc., F.R.Ae.S. (Member of Council of Royal Aeronautical Society); Mrs. Bulman; Group Captain P. W. S. Bulman, C.B.E., M.C., A.F.C., F.R.Ae.S.; Mrs. Bulman.

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Captain Andrew V. S. Yates, A.M.I.E.E.; R. W. Young, Esq.

1947 GARDEN PARTY

Although the number of tickets allowed to each member was more strictly limited than ever before, more than 6,000 people attended the Garden Party given by the Society on Sunday 14th September 1947 at the Handley Page Aerodrome, Radlett, Hertfordshire. Among the guests were many distinguished visitors from abroad, including a number of the American delegates to the Joint Aeronautical Conference.

The aerodrome was kindly lent for the occasion by the Ministry of Supply and Handley Page Ltd., and to them the Council wish to pay a warm tribute for making the aerodrome available and for their assistance in the general organisation of the Garden Party.

The Society of British Aircraft Constructors had held its annual Exhibition and Display at Radlett from the 9th to the 13th September and for the second year in succession generously agreed to keep the Exhibition and Display in being for the members of the Royal Aeronautical Society and their guests on Sunday the 14th.

The Council wish to put on record their sincere appreciation of the generosity of the

Society of British Aircraft Constructors and its members which made possible the holding of the Garden Party at that time.

Both the flying programme and the Static Exhibition arranged by the S.B.A.C. this year were on a bigger scale than ever. A total of forty-one of the latest British military and civil aircraft and five gliders were demonstrated in the air and another twenty-five aircraft were exhibited on the ground. In the Static Exhibition 188 firms had stands showing aircraft models and mock-ups, engines of all types, airscrews, parts of equipment and materials used in the production of British aircraft.

The Council wish to thank all those who so willingly contributed towards making the Garden Party the great success it was.

Group Captain A. F. Bandidt was in charge of the whole of the arrangements for the flying programme. The Council wishes to pay tribute to the efficient services of Group Captain Bandidt and his assistants and to thank the Ministry of Supply, the Royal Air Force, and the aircraft firms for their work in connection with the flying programme.

The following aircraft were demonstrated in the air:—

Handley Page Hermes II
Avro Tudor II
Handley Page Hastings I
Avro Lincoln II
Vickers-Armstrongs Valetta
Miles Marathon
de Havilland Dove
Bristol Freighter
Cierva W.9 Helicopter
Short Solent
Bristol Brigand
Miles Merchantman
Boulton Paul P. 108
Miles M.68
Fairey Firefly Trainer
Blackburn S.28/X3
Short Sturgeon
Hawker Sea Fury X
Gloster Meteor IV
de Havilland Vampire I
de Havilland D.H.108
Vickers-Supermarine Attacker
Vickers-Supermarine Seafire 47
Fairey Firefly IV
Westland Wyvern I
Airspeed Ambassador
Cunliffe-Owen Concordia
Elliott Newbury Eon
Portsmouth Aerocar
Percival Prentice T.1
Heston A.2/45 A.O.P.
Auster A.O.P. VI
Reid and Sigrist Desford
Percival Merganser
Avro Lincoln/Theseus
Avro Lancastrian/Ghost
Miles Aerovan
Auster Autocrat
de Havilland Chipmunk (shown by de Havilland Aircraft of Canada Ltd.)
Miles Gemini
Miles Messenger
General Aircraft Medium V Wing Form research Glider towed by H.P. Halifax
Slingsby Kirby Gull IV Sailplane towed by Auster
Short Nimbus Sailplane towed by D.H. Tiger Moth
Elliott Olympia Eon Sailplane towed by Elliott Newbury Eon
Martin Hearn Kite II Sailplane towed by Auster

The broadcasting system and loud speaker arrangements were provided by Rediffusion Ltd. and Group Captain H. J. Saker, M.V.O., A.F.R.Ae.S., broadcast the programme of events and necessary arrangements throughout the afternoon.

In addition to the R.A.F. Medical Officers, Dr. A. Buchanan Barbour, O.B.E., F.R.Ae.S. of British European Airways, was in attendance during the afternoon as Honorary Medical Officer.

During the afternoon the Fairey Aviation Works Band played a series of selections by courtesy of the Fairey Aviation Co. Ltd.

Besides the aircraft in the flying programme the following aircraft were shown on the ground:—

Airspeed Consul Convertible
Airspeed Consul
Blackburn Firebrand V
Bristol Buckmaster
de Havilland Sea Hornet XX
de Havilland Mosquito
de Havilland Dove
de Havilland Vampire III
Fairey Guided Missile
Fairey Gyrodyne Helicopter
Hawker Fury I (Napier Sabre)
Hawker Fury I (Bristol Centaurus XVIII)
Percival Proctor VI
Vickers Viking
Vickers-Supermarine Sea Otter
Vickers-Supermarine Spitfire Trainer
Percival Proctor IV (shown by Field Aircraft Services Ltd.)
Scottish Aviation A.4/45
de Havilland Leopard Moth (shown by Graviner Manufacturing Co. Ltd.)
General Aircraft Medium V Wing Form Research Glider
General Aircraft Maximum V Wing Form Research Glider
Slingsby Type 21 Sailplane
Elliotts of Newbury Primary Eon Glider
Elliotts of Newbury Baby Eon Glider
Martin Hearn Kirby Tutor Sailplane

The Council wish to thank sincerely all the pilots, firms, ground crews and others who so willingly helped with the display of all the aircraft, both those demonstrated and those exhibited on the ground.

The Council also wish to record their thanks and appreciation to the Royal Air Force and Peto and Radford Ltd. for providing starting facilities for the aircraft; to General Fire Appliances Ltd. for supplying fire appliances; the Royal Air Force and Handley Page Ltd. for supplying crash tender and fire service teams and other ground services; and to Alvis Ltd., Armstrong-Siddeley Ltd. and Rootes Securities Ltd. who kindly provided cars for the transport on the aerodrome of pilots and officials.

The Council also wish to place on record their appreciation of the help given by the Hertfordshire County Police; the British Red Cross Society (Hertfordshire); the Automobile Association; and the many firms and individuals who so readily rendered assistance.

The Static Display attracted great interest also and the Council wish to thank the following for their assistance in keeping open their stands during the afternoon and helping to make the Garden Party such a success:—

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REVIEWS

German Research in World War II. L. E. Simon. John Wiley & Sons Ltd., New York; Chapman & Hall, London. 1947. 24s. net.

This is not a book explaining how far ahead the Germans were in everything (except winning the war), and how extremely lucky the Allies were to be in the war at all as long as they were. It is an analysis of the conduct of research by Colonel Simon, who is the U.S. Army Director of the Ballistic Research Laboratories and was sent to Europe just before the German surrender to study German scientific establishments, their organisation and personnel. It contains brief accounts, which are of considerable interest, of much of the research work, but its real value lies in the author's description of the German research organisation, of the attitude of the German war leaders to scientific research for war purposes, and his own summary of the right conditions to foster research.

The successful functioning of German research failed largely because of the failure to plan basic research in conjunction with future requirements for development, and the lack of technical experts outside the various firms. An immense amount of research was done and some of it was bound to reach the stage of successful development. But the lack of direction at a high level led not only to waste of effort, but to the certain loss of the war. That does not mean that the Germans would have finally won, if the direction had been good, for the atomic bomb was too far advanced in the U.S.

Colonel Simon's last chapter "Comments and criticisms applicable to research in general," is one which should be most carefully read by all concerned with basic research, lecture research, design and development.

An excellent book.

Aircraft Engines of the World 1947. Paul H. Wilkinson. Sir Isaac Pitman & Sons Ltd., London, 1947. 50s. 0d. net.

This annual has now taken its place among the standard reference books which those engaged in the profession of aeronautics should have on their shelves. It gives complete specifications of all the latest aircraft engines in a standardised form, in both English and metric systems of measurement, together with a photograph of each engine. Particulars of the engines of eleven countries are given, from the small glider engines to the Lycoming XR-7755 of 5,000 h.p., from the Armstrong Siddeley Mamba to the V2, from those which are in production to those which are in the prototype stage. In the latter category, particulars are given of the Fedden Flat Six which leaves one with an uneasy feeling that the driving force and high ability of one of the really great British aero-engine experts are not being used at a time when every single ounce of effort is wanted.

AN APPRECIATION

The Council of the Royal Aeronautical Society wish to thank those Companies who, by their generous co-operation, have done so much to help in the production of the Journal

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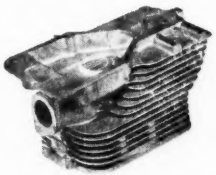

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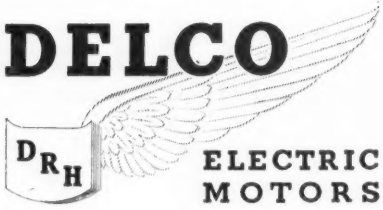

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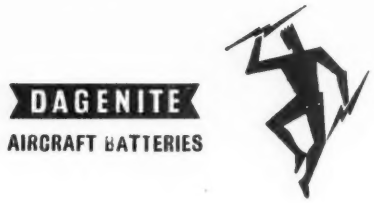

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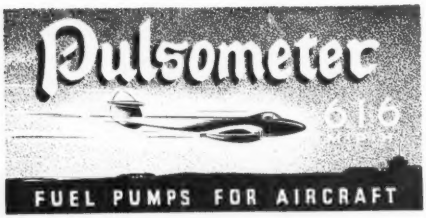

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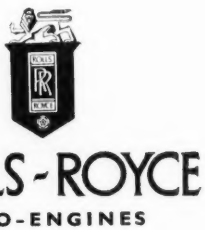

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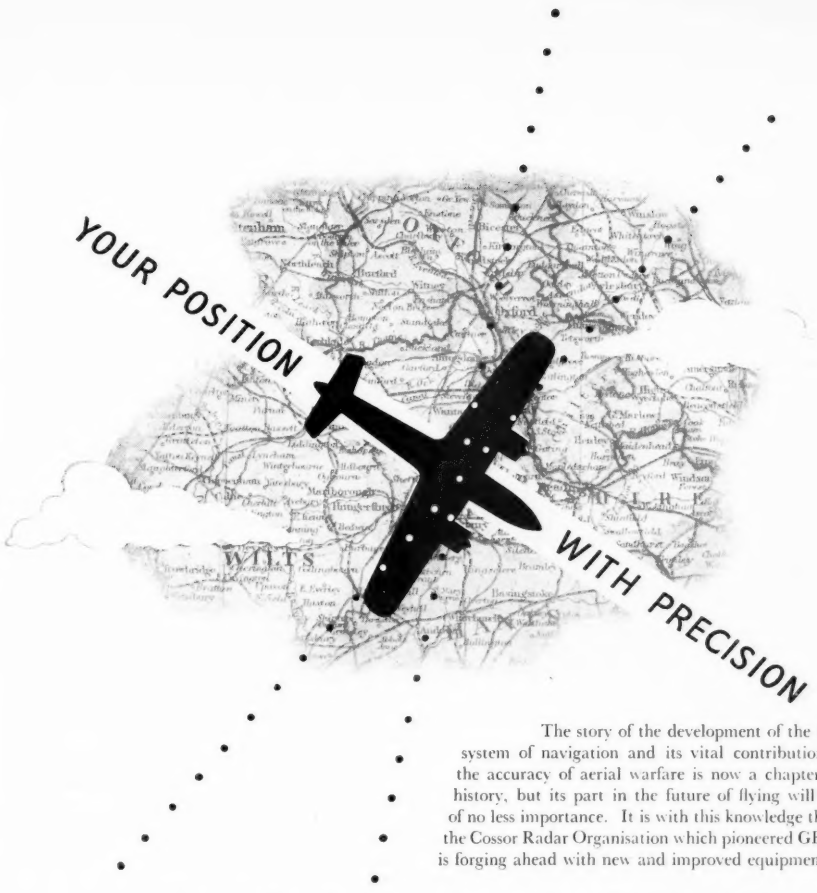

The story of the development of the GEE system of navigation and its vital contribution to the accuracy of aerial warfare is now a chapter in history, but its part in the future of flying will be of no less importance. It is with this knowledge that the Cossor Radar Organisation which pioneered GEE is forging ahead with new and improved equipment.

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Enquiries regarding GEE should be addressed to:—
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Printed by the LEWES PRESS (Wightman & Co. Ltd.), Friars Walk, Lewes, England, and Published by THE ROYAL AERONAUTICAL SOCIETY, 4 Hamilton Place, London, W.1, England.